Barely locally presentable categories

J. Rosický

joint work with L. Positselski

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If coproduct injections are monomorphisms then A is barely λ-presentable if and only if for every morphism $f : A \to \coprod_{i\in I} K_i$ there is a subset J of I of cardinality less than λ such that f factorizes as $A \to \coprod_{j\in J} K_j \to \coprod_{i\in I} K_i$ where the second morphism is the subcoproduct injection.

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Coproduct injections are very often monomorphisms, for instance in any pointed category. However, in the category of commutative rings, the coproduct is the tensor product and the coproduct injection $\mathbb{Z} \to \mathbb{Z} \otimes \mathbb{Z}/2 \cong \mathbb{Z}/2$ is not a monomorphism.

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Any locally λ-presentable category is barely locally λ-presentable.

A category $\mathcal{K}$ has λ-directed unions if for any λ-directed set of subobjects $(K_i)_{i\in I}$ of K the induced morphism $\operatorname{colim}_{i\in I} K_i \to K$ is a monomorphism. The following result was proved by Positselski and Šťovíček for abelian categories and for $\lambda = \aleph_0$.

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We say that $\mathcal{K}$ has *coproduct λ-directed unions* if for every coproduct λ-directed colimit $\coprod_{j \in J} K_j \to \coprod_{i \in I} K_i$, every morphism $\coprod_{i \in I} K_i \to K$ whose compositions with $\coprod_{j \in J} K_j \to \coprod_{i \in I} K_i$ are monomorphisms is a monomorphism.

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Proposition 2. Any barely locally λ-presentable category has coproduct λ-directed unions.

Proposition 3. Let $\mathcal{K}$ be a locally presentable category such that $\mathcal{K}^{\mathrm{op}}$ has coproduct λ-directed unions for some regular cardinal λ. Then $\mathcal{K}$ is equivalent to a complete lattice.

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Thus a non-trivial locally presentable category cannot have the barely locally presentable dual.

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Conversely, from the negation of Vopěnka's principle, we construct artificial examples of regular barely locally presentable categories which are not locally presentable.

Problem. Is there a barely locally presentable category which is not locally presentable in ZF?

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In fact, A is barely λ^+-presentable where λ is its uniform character (Hušek 1973), i.e., the smallest cardinality of a base of uniform covers of A.

$\mathbf{Prox}^{\mathrm{op}}$ is regular but not barely locally presentable because trivial proximity spaces (any two non-empty subsets are near) are not barely presentable in $\mathbf{Prox}^{\mathrm{op}}$ and

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Lemma 2. Let $\mathcal{K}$ be a barely locally presentable category with pullbacks such that coproduct injections are monomorphisms. Then any object of $\mathcal{K}$ is barely presentable.

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$\mathbb{R}$ is a cogenerator in $\mathbf{Prox}_0$ because any separated proximity space is a subspace of powers of $\mathbb{R}$. But it is not a strong cogenerator in $\mathbf{Prox}_0$ because strong monomorphisms in $\mathbf{Prox}_0$ are closed embeddings.

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Proposition 4. $\mathbf{Prox}_{\mathbb{R}}^{\text{op}}$ is barely locally $\aleph_1$-presentable.

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Any K in $\mathbf{Prox}_{\mathbb{R}}$ induces a realcompact topological space. The category of proximity spaces is isomorphic to the category $\mathcal{K}$ whose objects are triples (X, bX, f) where $f : X \to bX$ is an embedding of X to its compactification, i.e., f makes X a dense subspace of a compact space bX. Consider the functor $G : \mathcal{K}^{\mathrm{op}} \to \mathbf{Ring}^{\to}$ sending (X, bX, f) to the monomorphism $C(f) : C(bX) \to C(X)$ where $C(X)$ is the ring of continuous functions $X \to \mathbb{R}$. This makes $\mathcal{K}^{\mathrm{op}}$ isomorphic to a full subcategory of the category $\mathbf{Ring}^{\to}$ of morphisms of rings. Since the latter is locally presentable, Vopěnka's principle implies that $\mathcal{K}^{\mathrm{op}}$ is locally presentable.

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We do not know whether the local presentability of $\mathbf{Prox}_{\mathbb{R}}^{\mathrm{op}}$ depends on set theory.

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A uniform space is barely λ^{+}-presentable in $\mathbf{Unif}^{\mathrm{op}}$ where λ is its uniform character. This follows from the fact that any uniformly continuous mappings from a subspace of a product depends on λ many coordinates (Vidossich 1970). This is not true for proximity spaces (Hušek 1973).

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Let $\mathbf{Unif}_1$ be the full subcategory of $\mathbf{Unif}$ consisting of subspaces of powers of $\mathbb{R}$. Spaces from $\mathbf{Unif}_1$ are rather special, any has the uniform character $< \aleph_1$. Let $\mathbf{Unif}_{\mathbb{R}}$ be the full subcategory of $\mathbf{Unif}_1$ consisting of closed subspaces of powers of $\mathbb{R}$.

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Proposition 6. $\mathbf{Unif}_{\mathbb{R}}^{\mathrm{op}}$ is locally $\aleph_1$-presentable.

Proposition 7. Any barely locally presentable category with pullbacks is complete.

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Proposition 8. Let $\lambda_1 < \lambda_2$ be regular cardinals. Then any barely λ_1-presentable category is barely λ_2-presentable.

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Proposition 8. Let $\lambda_1 < \lambda_2$ be regular cardinals. Then any barely λ_1-presentable category is barely λ_2-presentable.

Proposition 9. Let $\mathcal{K}$ be a barely locally λ-presentable category and $\mathcal{C}$ be a small category. Then the functor category $\mathcal{K}^{\mathcal{C}}$ is barely locally λ-presentable.