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Two-dimensional Magma-Repository Interactions

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Two-dimensional simulations of magma-repository interactions reveal that the three phases –a shock tube, shock reflection and amplification, and shock attenuation and decay phase– in a one-dimensional flow tube model have a precursor. This newly identified phase "zero" consists of the impact of magma against the drift roof, after breakthrough into the drift, which results into a large pressure pulse. This initial pressure pulse against the tunnel roof can be estimated using one-dimensional shock reflection models in the vertical direction. After phase zero there is a large-pressure region near the breakthrough and the subsequent phases: a shock tube phase, a shock reflection phase at the closed end of the drift and an attenuation phase are similar as in the one-dimensional flow tube model. The flow along the drift then becomes nearly one dimensional due to the large length to diameter ratio of the drift. Most notably, the pressure reflection pulse at the closed tunnel end is (much) larger than the initial pressure pulse at the tunnel roof. The presented simulations are preliminary in that they are purely two dimensional. Improved values of pressure pulses and volume fluxes of magma will be calculated in the future by incorporating the proper three-dimensional effects into averaged two-dimensional flow area models. The model considered herein is a necessary precursor of these future models.

Keywords: magma-repository interactions, decompressing magma flows, hyperbolic equations, flow tube models

AMS Subject Classification: 93A30, 35L65, 35L15, 74A50

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1. Introduction

There is a probability of volcanic activity in the proposed nuclear waste repository site at Yucca Mountain in the range of $10^{-3} - 10^{-4}$ over the next 10^4 years (*e.g.*, Connor *et al.*, 2000). These relatively high rates raise important questions about the nature of any volcanic flows which may ensue when wet basaltic magma, ascending in a dike, meets a series of subsurface repository drifts. The aim of the present note is to present pilot simulations which extend previously developed models of magma-repository interactions (Bokhove and Woods 2000, 2001, hereafter BW) to two dimensions in order to more fully understand and quantify the potential nature of these interactions.

BW studied the ascent of relatively wet and therefore effectively compressible basaltic magma through a vertical dike which intersects a horizontal tunnel of comparable cross-sectional area to the dike and located about 400m below the surface. This process is a simplified representation of some aspects of the interaction of a basaltic fissure eruption through the planned Yucca Mountain waste-repository site. They assume that prior to breakthrough of the high-pressure magma in the dike (10-20 MPa), the tunnel is filled with air at atmospheric pressure (0.1 MPa), and examine the rapid decompression and flow which develops following breakthrough into the tunnel. The volatile phase in the magma is mainly water and is exsolved from the melt into bubbles as the mixture decompresses. BW use a model which provides an averaged one-dimensional picture of the flow. After breakthrough, a dominant magmatic shock eventually develops at the tunnel end, which propagates backwards towards the dike, see figure 1.

BW use a one-dimensional model of the averaged flow which implies that only pressure gradient along the flow tube are modeled. In particular the flow tube model around the intersection of the vertical dike and the horizontal tunnel could be improved by taking into account higher-dimensional and geometrical effects. The results and restrictions of the flow tube model lead to the following key remaining questions: i) How high are transient high pressure values and gradients along the repository drift? ii) In particular, does the development of high pressure shock reflection events at the end of the drift promote fracture and breakthrough to the surface far away from the original dike-drift intersections? iii) Alternatively, could the initial pulse of the complicated rarefaction-shock wave interaction on the roof directly after breakthrough also cause fracture to occur near the dike-drift intersection? iv) Are these pressure gradients large enough to cause the drift wall to fail? v) What effects do alternative dike-drift geometries have on the estimated pressure and flow geometries?

Questions i-iii will be addressed in this note in a two-dimensional dike drift geometry, which is a simplification of the actual three-dimensional dike drift geometry in Yucca Mountain. Future research will incorporate three-dimensional effects by properly averaging across or along the dike, and across the drift. The extra numerical requirements for including these three-dimensional effects into numerical two-dimensional models are relatively small. The two-dimensional models considered herein provide therefore a validation of the more complicated future models. Questions iv-v are relegated to the future as well, although our present results would already allow some inferences about drift-wall failure to be made.

Two-dimensional calculations are much more involved than the one-dimensional calculations in BW. Consequently, a large parameter study is unrealistic within the scope of work. Furthermore, the combined modeling of the air and multiphase magma with their separate equations of state is technically difficult due to the complicated evolution of the magma-air interface. Therefore the model presented herein will include the following simplification: magma and air are modeled as one magmatic fluid in which air is modeled as a high-volatile content magmatic gas. The drift is thus filled initially with a magmatic gas at atmospheric pressure. A comparison of one-dimensional calculations for a magmatic fluid only with our full one-dimensional magma-air calculations shows that the initial shock amplification phase is more violent by a factor of 3-5 in the latter case when magma and air are modeled side by side. Nevertheless, the dynamics and final state in both approaches are qualitatively and to a lesser extent quantitatively similar.

2. Equations of motion

The magma equations of motion in a connected three-dimensional volume averaged in the lateral y-direction with a well-defined width $b = b(x, z)$ depend on the coordinates

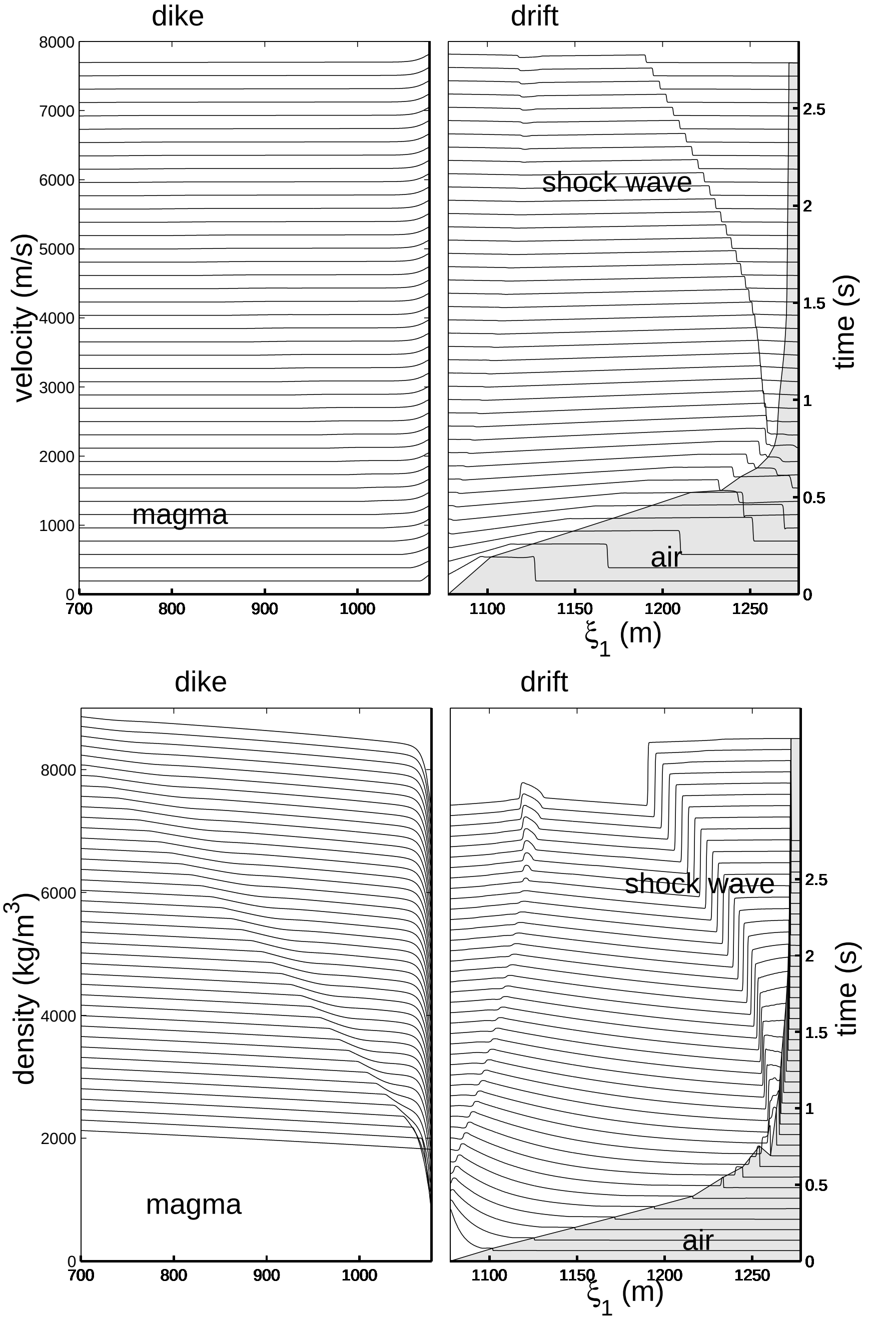


FIGURE 1. Velocity, density and pressure profiles are shown for magma-air interactions in a dike-drift system. The simulation encompasses $2.734\,s$ and each of the 41 profiles is spaced $0.0683\,s$ apart. Profiles in dike and drift have been separated to emphasize the different scaling in dike and drift. The rarefaction wave in the dike, the magma, the air, and the shock wave have been indicated. Subsequent profiles have been shifted upward with a fixed amount, which is somewhat reflected in the vertical time axis on the right, while the scale of the relevant quantity is found on the left. Short-time reference simulation 101. Copied with permission from BW.

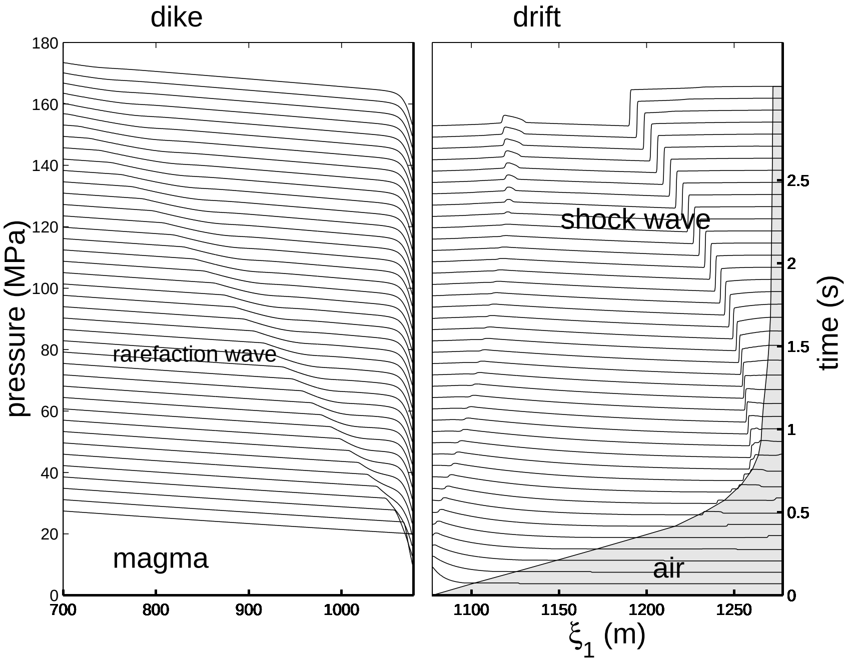


FIGURE 1. Continued, short-time reference simulation 101.

x, z in the vertical plane and time t, as follows:

$$\begin{aligned}\frac{\partial(\rho\,b\,u)}{\partial t}+\frac{\partial(\rho\,b\,u^2+p\,b)}{\partial x}+\frac{\partial(\rho\,b\,u\,w)}{\partial z}&=p\,\frac{\partial b}{\partial x}-F_1,\\ \frac{\partial(\rho\,b)}{\partial t}+\frac{\partial(\rho\,u\,b)}{\partial x}+\frac{\partial(\rho\,w\,b)}{\partial z}&=0,\\ \frac{\partial(\rho\,b\,w)}{\partial t}+\frac{\partial(\rho\,b\,u\,w)}{\partial x}+\frac{\partial(\rho\,b\,w^2+p\,b)}{\partial z}&=p\,\frac{\partial b}{\partial z}-g\,\rho-F_3,\end{aligned}\tag{2.1}$$

where u, w are the velocities in the x, z-directions, p is the pressure, ρ is the density, g is the gravity and the dissipative terms are denoted by $\mathbf{F} = (F_1, F_3)$. The multiphase basaltic fluid of melt and volatiles is modeled as an isothermal compressible fluid with a parameterized equation of state. The density ρ of this fluid equals the reciprocal of the volume occupied by a unit mass of the mixture of exsolved volatiles (gas), dissolved volatiles (liquid) and melt. The mass fraction of exsolved volatiles is $n(p)$. Dissolved volatiles and melt are lumped together as an incompressible mixture of mass fraction $1 - n(p)$ and density σ, giving

$$\rho(p)=\left(\frac{n(p)\,R_v\,T}{p}+\frac{1-n(p)}{\sigma}\right)^{-1},\tag{2.2}$$

where $R_v \approx 462\,J\,kg^{-1}\,K^{-1}$ is the gas constant for H_2O in the basaltic mixture, the mixture temperature $T = 1000 - 1200\,K$, and lumped melt-liquid density $\sigma \approx 2500\,kg\,m^{-3}$. Volatiles, bubbles and melt are assumed to be in chemical equilibrium modeled using Henry's law [Sparks, 1978]

$$n(p) = n_0 - n_{s_H} \equiv n_0 - s_H\,p^{\beta}.\tag{2.3}$$

The above model of multiphase magma follows the literature on explosive volcanic eruptions [*e.g.*, Wilson *et al.*, 1980; Woods, 1995].

A non-dimensional formulation of the equations of motion (2.1) with equation of state (2.2) and (2.3) are discretized using shock-capturing numerical techniques since the inertial (frictionless) terms dominate in the first few seconds after breakthrough due to the large initial pressure difference between dike and drift (*e.g.*, BW). The two-dimensional numerical models considered employ the method of lines and are therefore straightforward extensions of the one-dimensional numerical models used in BW. In this note, frictional effects are neglected, *i.e.* $\mathbf{F} = 0$.

3. Numerical results

Inviscid magma dynamics is investigated within two connected rectangular domains, *i.e.* the two blocks right of the dashed symmetry line in figure 6a. The connection between the two domains is a horizontal strip at the bottom left corner of block 1, the “drift” and the top of block 2, the “dike”. This configuration arises as a lateral (y-direction) average of the dike-drift geometry. In this section the geometrical factor $b(x, z) = 1$ in (2.1), that is, we are considering purely two-dimensional simulations. The implication is that the volume fluxes can only be compared qualitatively with the three-dimensionally averaged one-dimensional simulations results.

Simulations 212–214 are computed on an irregular grid with extra resolution around the dike-drift breakthrough. The boundary conditions in the three simulations are as follows: the West, North, East boundaries of block 1 are impenetrable, part of the South boundary of block 1 is impenetrable, the East and West boundary of block 2 is impenetrable, the South boundary of block 2 is open, and the North boundary of block 2 has an open connection with the bottom-left boundary of block 1.

The simulations in figure 2 shows the dynamics after the magma breaks from dike into drift every $0.0104\,s$ till time $0.1664\,s$. The one-dimensional reference simulation in figure 1 displayed three phases: the initial shock tube phase, the shock reflection phase, and finally the emerging attenuation phase in which rarefaction and shock waves leave the domain and decay. These two-dimensional simulations show that there is an additional phase in which a vertical magma jet forms and reflects against the roof as in a one-dimensional shock tube with reflection against a horizontal wall. Initially and near the symmetry line or wall on the left the speeds are high and the flow is more or less one dimensional. Due to the low-pressure region in the drift, the flow eventually becomes two dimensional, but further in the drift we can see in figure 3 that the flow becomes fairly one-dimensional again since the diameter of the drift is small relative to its length. Simulation 213 has a simple extrapolating boundary condition on the South boundary of block 1, thus allowing for waves to leave the domain. This boundary condition works reasonably well but does contaminate the interior, especially in combination with the nonzero value of gravity. This effect is very slow and the downward propagating rarefaction waves eventually diminish the effect completely. To check this we show pressure contour plots in figure 4 of the same simulation 213 but now with a solid South boundary in block 1: we see that there is no effect by changing the boundary. Hence the contamination around the South boundary is kept at bay from the key dike-drift areas of interest.

To assess the validity of the one-dimensional flow tube model, we show the pressure evolution of a one-dimensional model which is an average of the previous two-dimensional model. The connection at the breakthrough is smooth in contrast to the contract nozzle connection we considered in figure 1. When we compare figure 3 with figure 5, we see that in the two-dimensional calculations it takes time for the magma to turn around the corner and become nearly one-dimensional again: the reflection process in the two-dimensional simulation starts at time $1.04\,s$ while it begins at $0.65\,s$ in the one-dimensional case.

4. Summary and Discussion

Two-dimensional simulations of magma-repository interactions have identified a phase zero. The mathematical model consists initially of high pressure magma in the dike and a magmatic gas at atmospheric pressure in the drift, both modeled for simplicity with the same equation of state of a multiphase magma modeled as an effectively compressible fluid consisting of water vapor bubbles and liquid magma. Phase zero consists of the impact of magma against the drift roof, after breakthrough into the drift, which results into a large pressure pulse. This initial pressure pulse against the tunnel roof can be estimated using one-dimensional shock reflection models in the vertical direction. After phase zero there is a large-pressure region near the breakthrough and the subsequent phases: a shock tube phase, a shock reflection phase at the closed end of the drift and an attenuation phase are similar as in the one-dimensional flow tube model. The flow along the drift then becomes nearly one dimensional asymptotically due to the large length to diameter ratio of the drift. Most notably, the pressure reflection pulse at the closed tunnel end is (much) larger than the initial pressure pulse at the tunnel roof. The presented simulations are preliminary in that they are purely two dimensional. Improved values of pressure pulses and volume fluxes of magma will be acquired in the future by incorporating the proper three-dimensional effects into averaged two-dimensional flow area models. The model considered herein is a necessary precursor of these future models.

Since we don't model the feedback of the time- and space dependent magma pressure on the rock or vice versa explicitly, we have to prescribe the dike geometries as functions of space and time. Through this geometric description, the dike walls have an influence on the magma fluid dynamics in the dike and drift. Various possible dike-drift geometries under consideration are shown in figure 6, corresponding to the bulbous dike-tip model of Lister [1990] and the crack-tip model of Rubin [1993, 1995]. Our numerical model is restricted to geometries that consist of multiple, connected, weakly-deformable rectangular blocks, because there must exist a transformation to a rectangular domain in each computational block. Moreover, the resulting geometrical factors must not place to much restriction on the numerical performance. Only configurations figure 6a, b, d, and figure 6e are therefore acceptable. In principle, we can also consider asymmetric dike-drift intersections, which will of course require more computational effort.

The presented research constitutes the interim report of contract research on two-dimensional magma-repository interactions performed by the author for the Center of Nuclear Waste Regulatory Analyses, Southwest Research Institute, San Antonio, Texas, U.S.A. It is a pleasure to thank the staff of Southwest Research Institute for all their hospitality during my various visits.

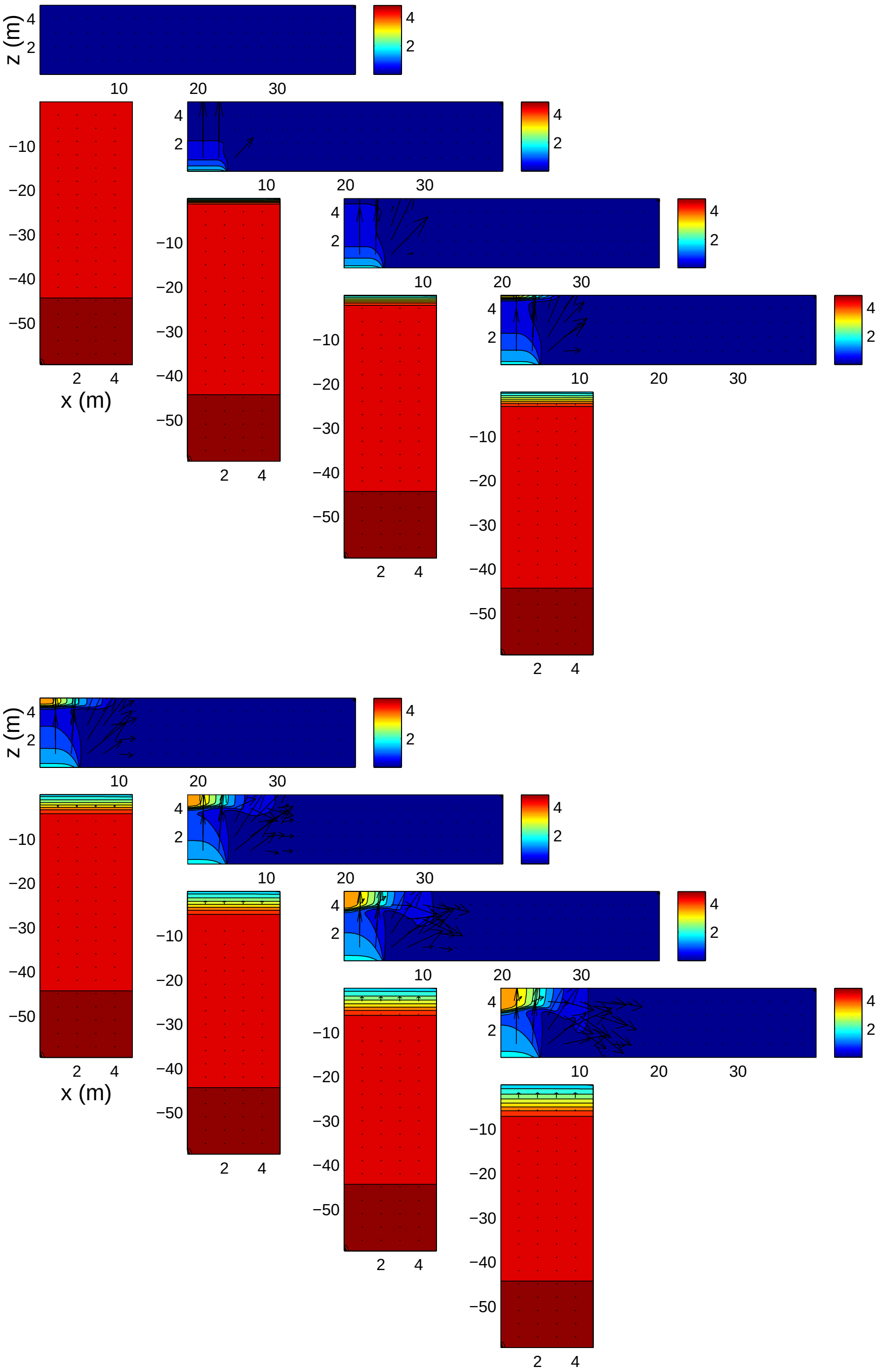
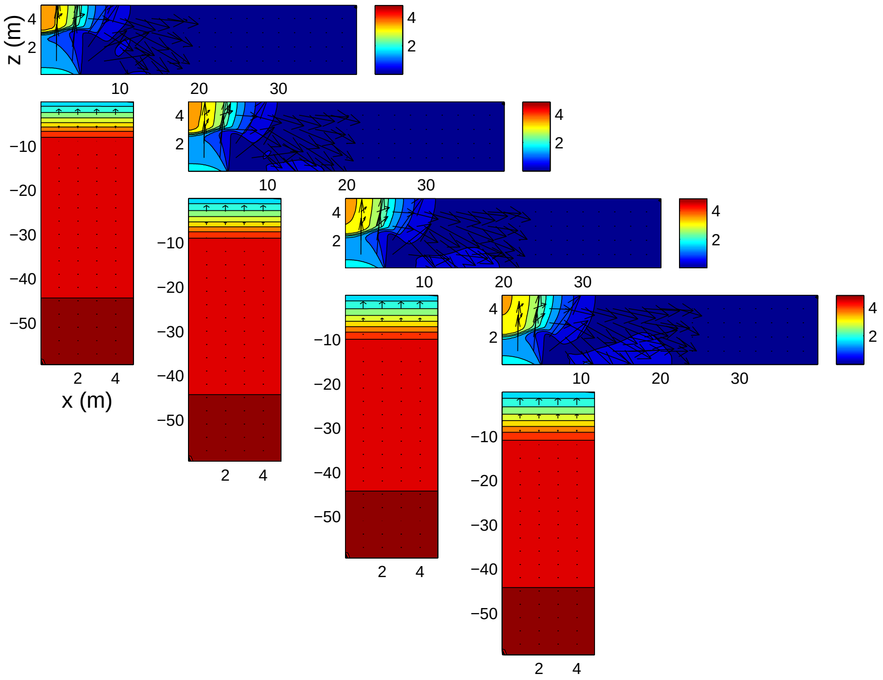


FIGURE 2. Shock tube with hydrostatic initial condition in dike and drift, *e.g.*, simulation with $g \neq 0$. Pressure contours in MPa are shown. Arrows indicate the flow direction. Time spacing is $0.0104\,s$. Run 212.


z (m)
4
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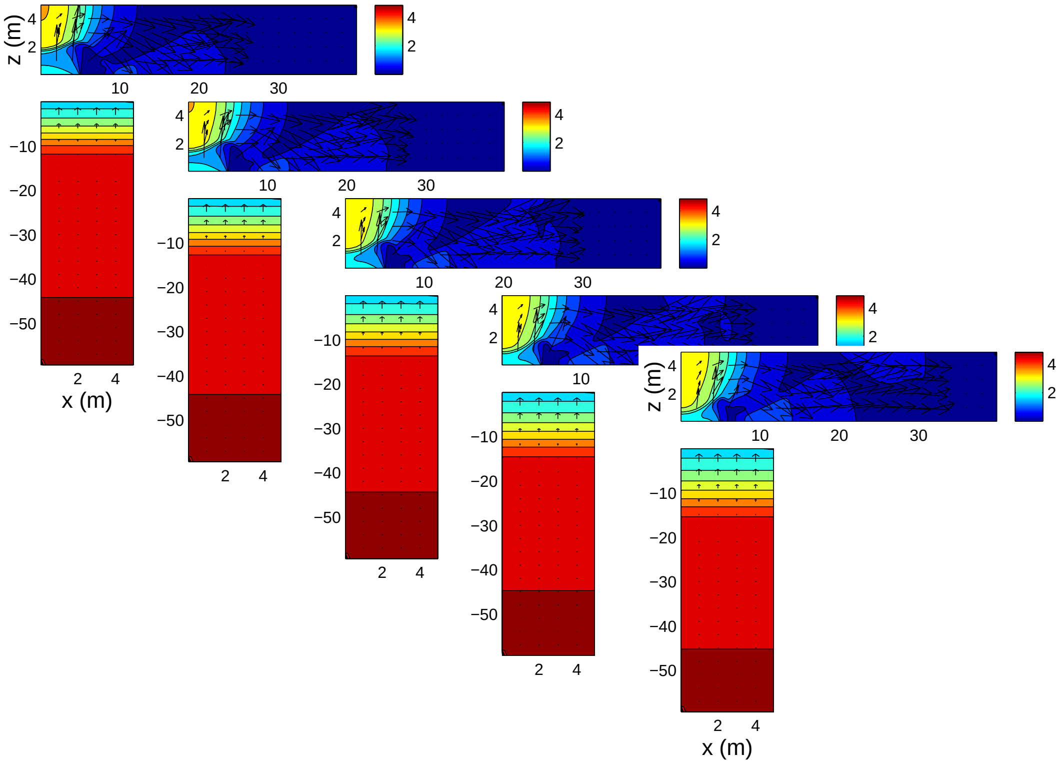
2
4
x (m)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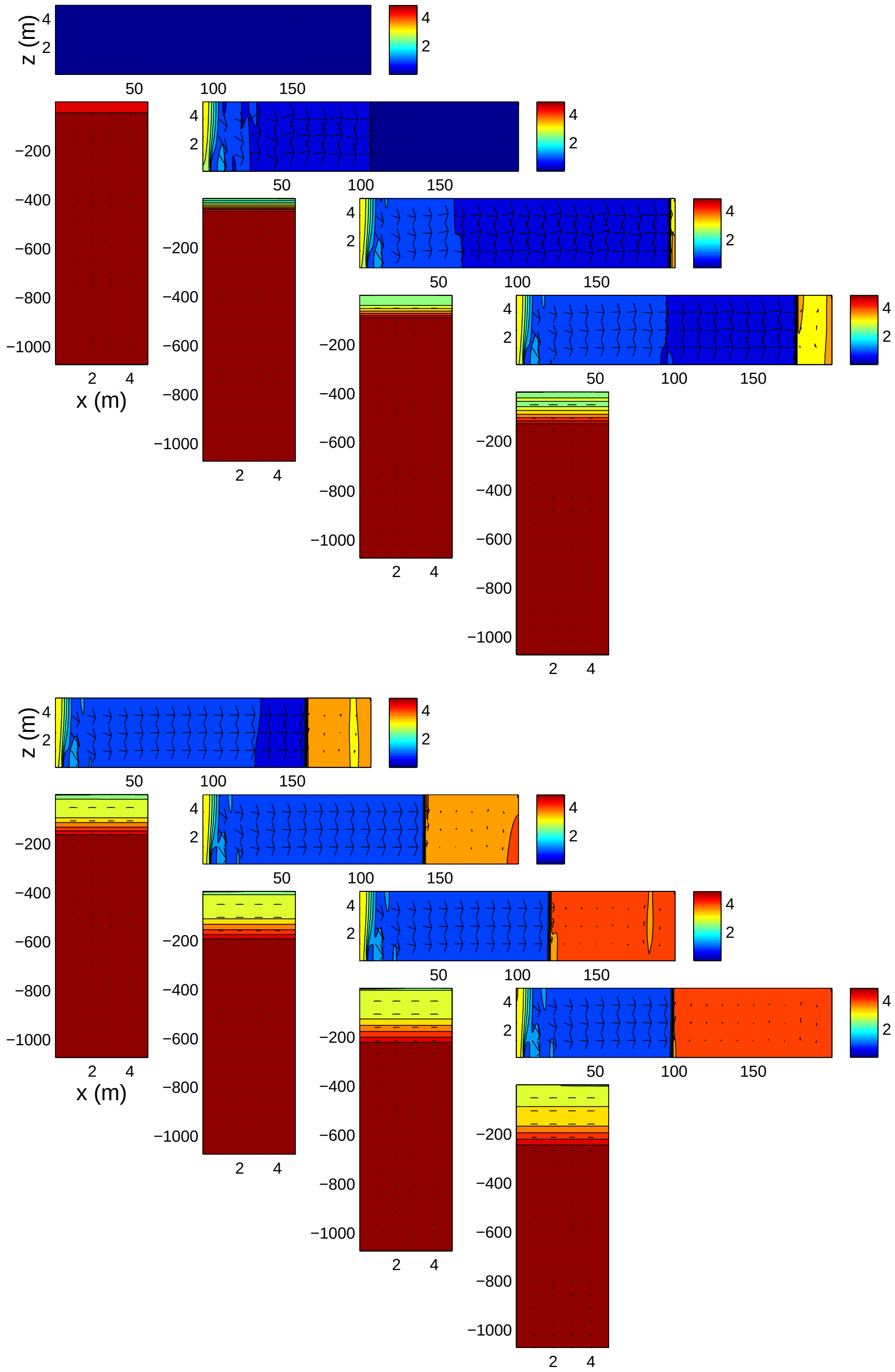

FIGURE 3. Shock tube with hydrostatic initial condition in dike and drift, *e.g.*, simulation with $g \neq 0$. Pressure contours in MPa are shown. Time spacing $0.5202\,s$. Run 213.

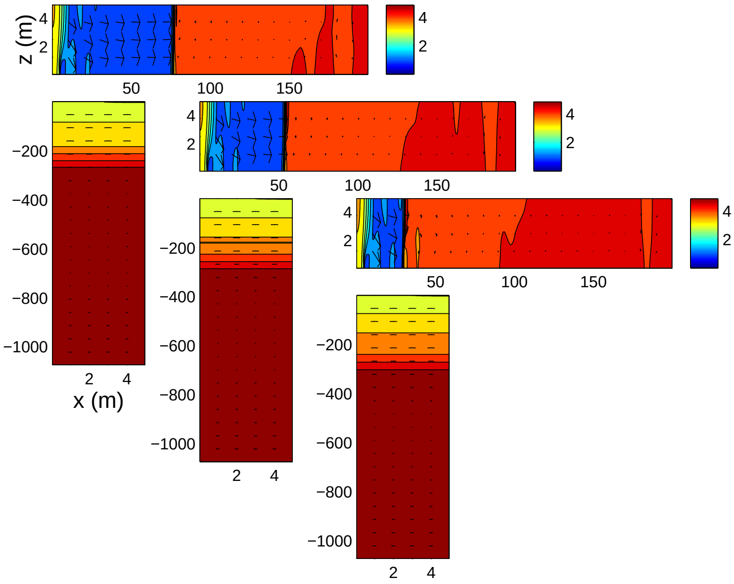

FIGURE 3. 213 continued.

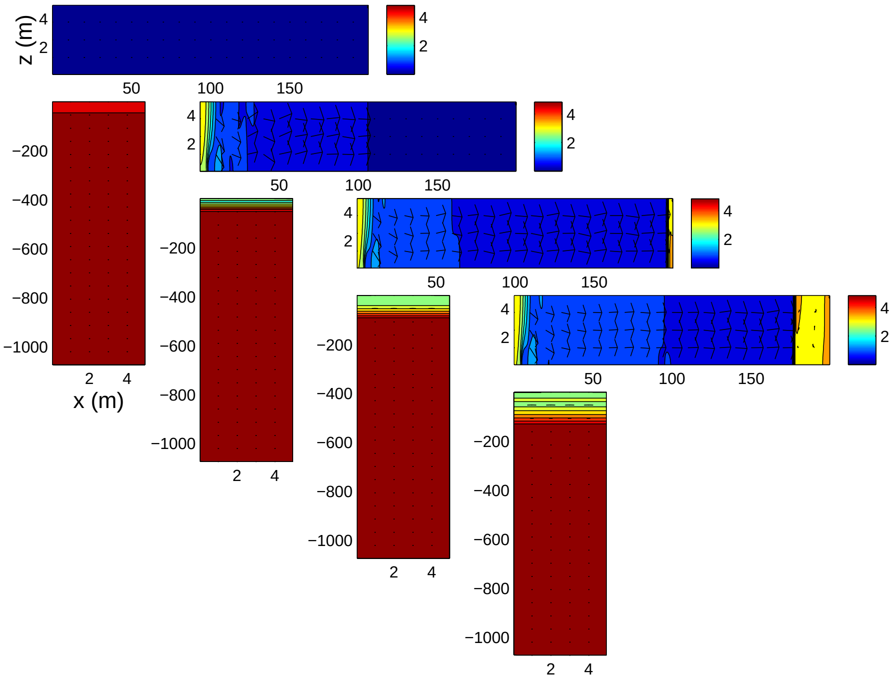

FIGURE 4. Shock tube with hydrostatic initial condition in dike and drift, *e.g.*, simulation with $g \neq 0$. Pressure contours in MPa are shown. Time spacing $0.5202\,s$. Run 214.

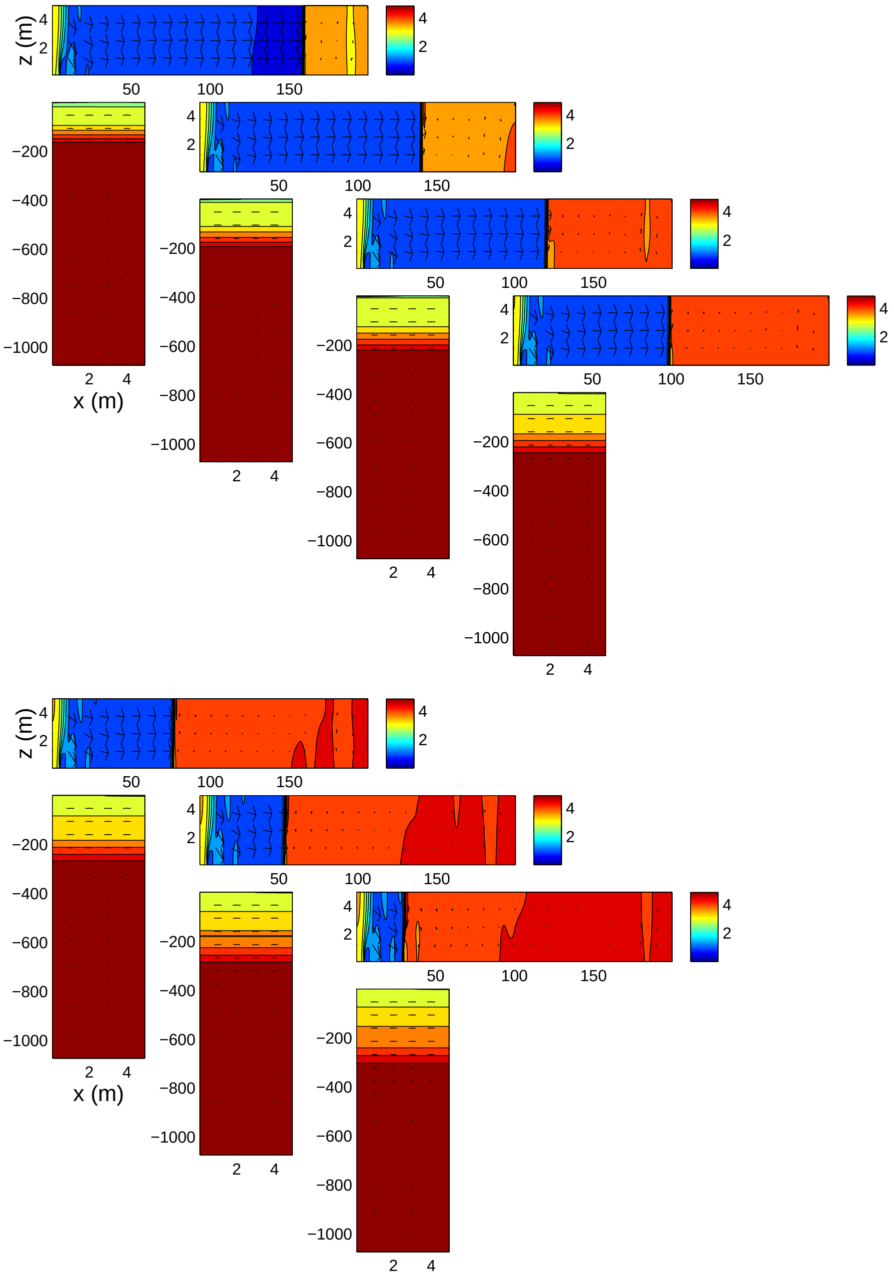

FIGURE 5. 214 continued.

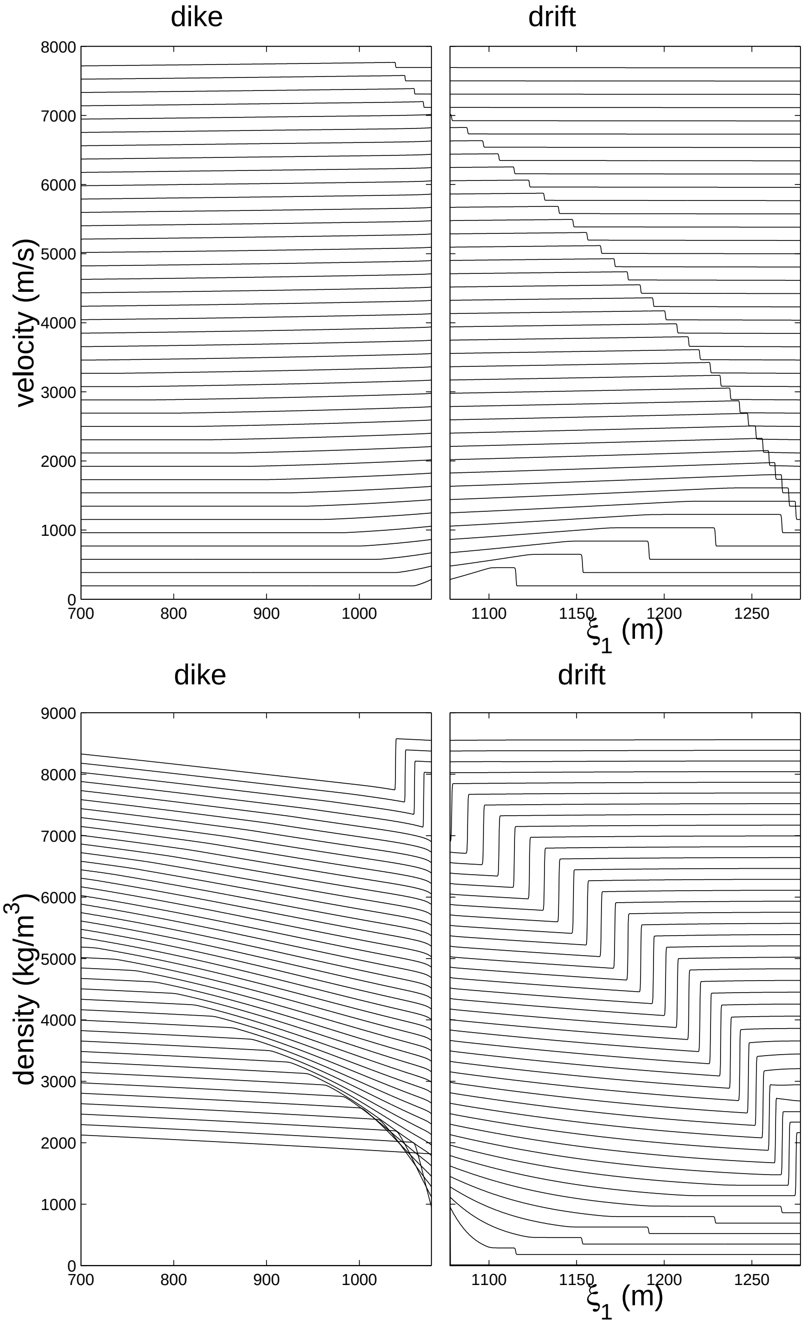

FIGURE 5. Velocity, density and pressure profiles are shown for magma flow in a dike-drift system. The one-dimensional model is an average over the two-dimensional geometry. Spacing $0.13\,s$.

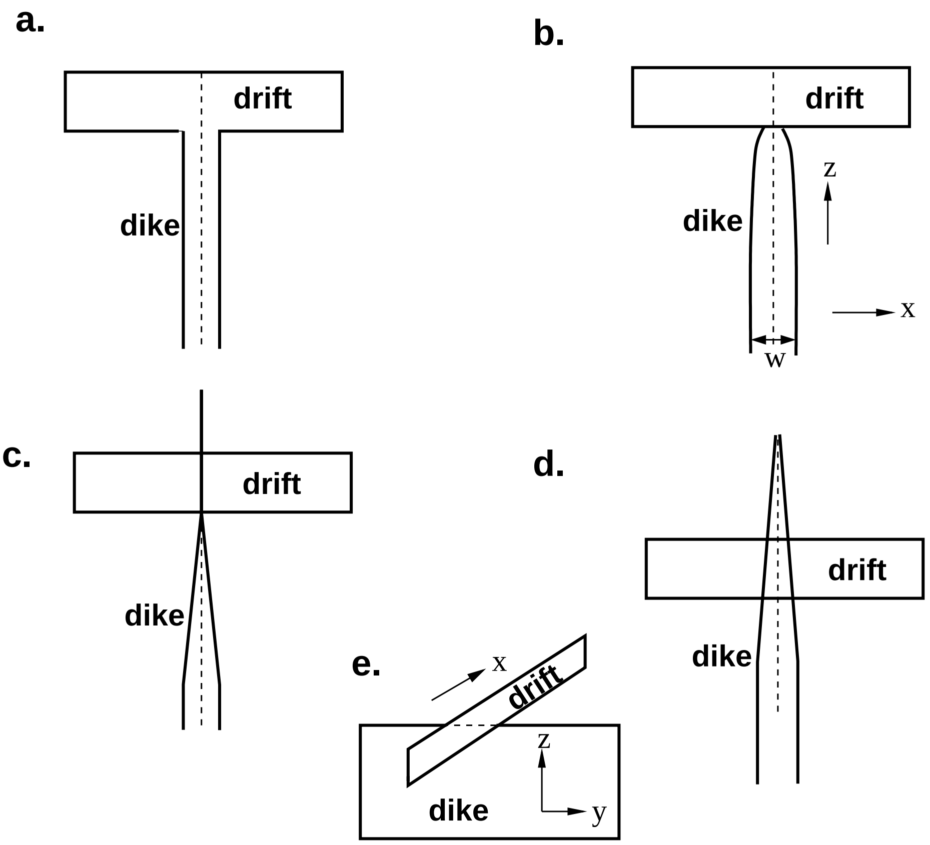

Figure 6. Various dike drift geometries under consideration are: laterally averaged models such as (a) the standard reference case, (b) a bulbous dike tip model, (c) a thin crack model, and (d) a thin crack model at a later time, and (e) a mixed model laterally averaged in the drift and averaged in the x-direction across the dike. The width $w = w(z,t)$ can be a function of time as long as the flow in the computational domain can be modeled reasonably well as two or three weakly deformed rectangular blocks. The dashed vertical line denotes the line of symmetry.

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