UG/PG student project in applied mathematics, 2014-15

Supervised by Dr Mauro Mobilia (Room 10.13, email: M.Mobilia@leeds.ac.uk, Phone: 31591)

Title: "Evolutionary Dynamics of Rock-Paper-Scissors Games"

Project suitable for undergraduate and postgraduate students (3rd-4th year and MSc students)

Introduction & Motivation: Understanding the maintenance of biodiversity and the emergence of cooperation is an important topic in life and behavioural sciences (e.g. biology, ecology and economics). Evolutionary game theory, where the success of one species depends on what the others are doing, provides a promising mathematical framework to study the coexistence and collective dynamics of populations [1,2]. In particular, rock-paper-scissors games - in which rock crushes scissors, scissors cut paper, and paper wraps rock - have emerged as a fruitful metaphor for non-hierarchical co-evolutionary dynamics [3-5] as observed in some recent experiments on microbial communities [5].

Aims of the proposed project:
In Refs. 3-5 the cyclic competition among three strains of E.coli bacteria was idealized assuming symmetric and short-range interactions in the absence of any form of mutations. Important aspects of cyclic evolutionary dynamics are revealed by investigating the properties of the system's replicator equations, which a set of ordinary differential equations [1], and the influence of noise by adopting an individual-based approach. While the expectations will be different, for 3rd-4th year students and MSc students, the general goals of this project are the following:

- To study the replicator equations of the rock-paper-scissors game (and its variants) in the absence/presence of mutations. In particular, the cases where interior fixed points, limit cycles (Hopf bifurcation) and heteroclinic cycles shall be analysed both analytically and numerically.

- Depending on the progress, and if time allows, to follow Refs. [3,6,7] and formulate an individual-based stochastic description of the above models to assess some effects of stochastic noise on the evolutionary dynamics in a finite (well-mixed) population.

With this project, the student will become acquainted with basic concepts of evolutionary game theory (payoff, fitness, replicator dynamics), stochastic methods and nonlinear dynamics. The student will also have the opportunity to become familiar with the probabilistic formulation of a broad range of problems (Markov processes, diffusion approximation, 1st-passage problems) and will be able to give a multidisciplinary interpretation of the models studied.

Nature of the project: ~70 % analytical (master equation, stochastic calculus, differential equations) and ~30 % numerical (solving ODEs and stochastic simulations).

Prerequisites, Commitments and Assessment:: see overleaf for detailed information

Possible developments: This project can serve as a starting point for further investigations, e.g. the influence of spatial degrees of freedom and pattern formation.

References:
1. J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics*.
2. M. A. Nowak, *Evolutionary Dynamics* (Belknap Press, Cambridge, 2006).
3. T. Reichenbach, M. Mobilia and E. Frey, Physical Review E **74**, 051907 (2006), Nature **448**, 1046 (2007); Phys. Rev. Lett. **99**, 238105 (2007).
4. M. Mobilia, Journal of Theoretical Biology **264**, 1 (2010).
5. B. Kerr et al., Nature **418**, 171 (2002).
6. R. Erban, J. Chapman, and P. Maini, arXiv:0704.1908v2 (http://arxiv.org/pdf/0704.1908.pdf).
7. C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983).

Supervision: This project and lines of investigation have been proposed and will be supervised by Dr Mauro Mobilia (Email: M.Mobilia@leeds.ac.uk, phone: 31591).

UG/PG Applied Project on "Evolutionary Dynamics of Rock-Paper-Scissors Games":
Prerequisites, Commitments and Assessment Criteria

Project suitable for undergraduate and postgraduate students

Prerequisites: for this project it is required and assumed that the students have a

- ***Good knowledge* of *calculus* and *linear algebra***
- ***MATH1920 : Good knowledge* of *scientific computing (Mathematica, Matlab, …)***
- ***MATH2391 : Good knowledge of the theory of nonlinear differential equations***
- ***MATH2750: Good knowledge of probability and stochastic (Markov) processes.***

The student's commitments:

- According to the module catalogue: **142 hours of private study** for MATH3422/MATH3423, and respectively **287 and 383 hours** of private study for MATH5003 and MATH5004.
- To punctually attend all meetings, being duly prepared, with the supervisor
- To swiftly inform the supervisor if s/he cannot attend a meeting and/or of any changes concerning the previously agreed work-plan
- To work regularly by himself/herself on the project, as instructed by the supervisor.
- To search the literature and do the relevant reading
- To write an independent maths report on the project

The supervisor's commitments:

- According to the module catalogue: **8 hours** of supervision meetings for MATH3422/MATH3423, and respectively **13 and 16 hours** of supervision for MATH5003 and MATH5004.
- To provide scientific guidance, e.g. concerning tasks and relevant literature
- To punctually attend all meetings
- To swiftly inform the student if he cannot attend a meeting and/or of any changes in what had been previously planned

What the student can expect from the supervisor:

- To provide scientific guidance, e.g. concerning tasks and relevant literature
- Regular supervision meetings
- Scientific guidance
- Reference to the appropriate literature
- Fair assessment and appropriate feedback

What the student cannot expect from the supervisor:

- The supervisor will not write any parts of the student's report
- The supervisor will not perform any calculations for the student
- The supervisor will not search the literature, order books or articles for the student.

Assessment: see the module catalogue. *There will be an oral presentation of the project (oral assessment or viva) and a discussion of the report and of its presentation. The main assessment criterion concerns the mathematical understanding and ability demonstrated in the report and in its oral presentation.* The assessment will take into account how much of the original project was eventually covered, and the understanding demonstrated by the student.