UG/PG student project in applied mathematics, 2014-15

Supervised by Dr Mauro Mobilia (Room 10.13, email: M.Mobilia@leeds.ac.uk, Phone: 31591)

Title: *"Dynamics of social dilemmas: cooperation and defection in evolutionary games"*

Project suitable for undergraduate and postgraduate students (3rd-4th year and MSc students)

Motivation and project description: Understanding the emergence of cooperation is a topic of paramount importance in life and behavioural sciences. In fact, the emergence of cooperation seriously challenges Darwinian evolution since altruism increases the fitness of a population at a cost for the single individual. Therefore, in the absence of appropriate mechanisms, cooperation is not sustained by evolution. Nevertheless, cooperative behaviour abounds in nature. In this context, evolutionary game theory, where the success of one species depends on what the others are doing, provides an ideal mathematical framework to model collective dynamics of interacting populations and study the origin of cooperation [1-3]. The traditional approach to study evolutionary dynamics of games is through the celebrated replicator equations [1-3] that are a set of nonlinear ordinary differential equations [1-3]. While the predictions of such a deterministic approach has proved insightful, it has long been recognized that stochastic effects due to demographic fluctuations (populations are of finite size) often have drastic consequences on evolutionary dynamics [2,4,5].

Aims of the proposed project: The goal of this project is to study the dynamics of social dilemmas in terms of evolutionary games, where each player can adopt simply two strategies: either to "cooperate" or to "defect". Depending on the detailed interactions between the different players, one distinguishes different classes of games ("prisoner's dilemma", the "stag-hunt" and the "snowdrift") characterized by bistability , the coexistence of both strategies, or the dominance of one strategy [1-3]. In this project, we shall consider the "prisoner's dilemma", the "stag-hunt" and the "snowdrift" games as prototypical models of cooperation dilemmas and study their dynamical properties using the replicator equations as well as a stochastic description. While the expectations will be different, for 3rd-4th year students and MSc students, the general goals of this project are the following:

- The deterministic dynamics of the "prisoner's dilemma", the "stag-hunt" and the "snowdrift" games will be understood. The replicator equations could then be generalized to account for mutations
- The evolutionary dynamics of these games in well-mixed and finite populations will be investigated in terms of Markov processes [2,4,5] by analytical means complemented by stochastic simulations (Gillespie algorithm [6]). We shall be particularly interested in the probability that a given strategy is adopted by the entire population (it "fixates") and the mean time for this to occur.

Nature of the project: ~70 % analytical (master equation, stochastic calculus, differential equations) and ~30 % numerical (solving ODEs and stochastic simulations).

Prerequisites, Commitments and Assessment: see overleaf for detailed information.

References:

1. J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics*.
2. M. A. Nowak, *Evolutionary Dynamics* (Belknap Press, Cambridge, 2006).
3. H. Gintis, Game Theory Evolving (Priceton, 2nd Edition, 2009)
4. C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983).
5. S. Karlin & H. M. Taylor, *A second Course in Stochastic Processes* (Academic Press, 1981).
6. R. Erban, J. Chapman, and P. Maini, arXiv:0704.1908v2 (http://arxiv.org/pdf/0704.1908.pdf).

Supervision: This project and lines of investigation have been proposed and will be supervised by Dr Mauro Mobilia (Email: M.Mobilia@leeds.ac.uk, phone: 31591).

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Project suitable for undergraduate and postgraduate students (3rd-4th year and MSc students)

Prerequisites: for this project it is required and assumed that the students have a

- ***Good knowledge* of *calculus* and *linear algebra***
- ***MATH1920 : Good knowledge* of *scientific computing (Mathematica, Matlab, …)***
- ***MATH2391 : Good knowledge of the theory of nonlinear differential equations***
- ***MATH2750: Good knowledge of probability and stochastic (Markov) processes.***

The student's commitments:

- According to the module catalogue: **142 hours of private study** for MATH3422/MATH3423, and respectively **287 and 383 hours** of private study for MATH5003 and MATH5004.
- To punctually attend all meetings, being duly prepared, with the supervisor
- To swiftly inform the supervisor if s/he cannot attend a meeting and/or of any changes concerning the previously agreed work-plan
- To work regularly by himself/herself on the project, as instructed by the supervisor.
- To search the literature and do the relevant reading
- To write an independent maths report on the project

The supervisor's commitments:

- According to the module catalogue: **8 hours** of supervision meetings for MATH3422/MATH3423, and respectively **13 and 16 hours** of supervision for MATH5003 and MATH5004.
- To provide scientific guidance, e.g. concerning tasks and relevant literature
- To punctually attend all meetings
- To swiftly inform the student if he cannot attend a meeting and/or of any changes in what had been previously planned

What the student can expect from the supervisor:

- To provide scientific guidance, e.g. concerning tasks and relevant literature
- Regular supervision meetings
- Scientific guidance
- Reference to the appropriate literature
- Fair assessment and appropriate feedback

What the student cannot expect from the supervisor:

- The supervisor will not write any parts of the student's report
- The supervisor will not perform any calculations for the student
- The supervisor will not search the literature, order books or articles for the student.

Assessment: see the module catalogue. *There will be an oral presentation of the project (oral assessment or viva) and a discussion of the report and of its presentation. The main assessment criterion concerns the mathematical understanding and ability demonstrated in the report and in its oral presentation.* The assessment will take into account how much of the original project was eventually covered, and the understanding demonstrated by the student.