

Undergraduate student project in applied mathematics, 2014-15
Supervised by Dr Mauro Mobilia (Room 10.13, email: M.Mobilia@leeds.ac.uk, Phone: 31591)

Title: “Opinion dynamics with the voter models”

Project suitable for undergraduate and summer bursary students

Motivation and Project description

Modelling complex systems using stochastic methods and nonlinear dynamics has provided significant insight into interdisciplinary problems like the emergence of collective phenomena [1]. One paradigmatic example in the realm of social dynamics is the two-state "voter model", where individuals in a population can be in one of two opinion states [1]. In this model, an individual is selected at random and adopts the state of one of its randomly-chosen neighbour; this update step is applied repeatedly until the dynamics ceases because a consensus is reached (with all agents sharing the same "opinion"). The voter model is also closely related to evolutionary games used to study cooperation dilemmas and can readily be generalized to more than two states [1-3]. Questions of great interest concern the emergence and maintenance of multicultural diversity and the circumstances under which a consensus is attained. In this project, it is proposed to study some properties of the two-state voter model and its generalizations by analytical and numerical means.

Aims of the proposed project

The detailed expectations will be different for 3rd-4th year students and for summer bursary students, but the main goals of the project are the following:

- (i) The student will investigate the probability that the classic two-state voter model ends in a specific consensus state and the mean time for such an event to occur.
- (ii) It will be studied how the presence of a fraction of committed individuals ("zealots") affects the dynamics of the classic voter model [2]. Is a consensus still possible? Is there a coexistence state ?
- (iii) A three-state generalization of the voter model where two states (say "rightists" and "leftists") are incompatible and interact with a third state ("centrists") to impose their consensus will be considered. It will be shown that in this case the dynamics can settle in a polarized state consisting only of leftists and rightists [3]. The probability that such an event occurs will be investigated.

References

- [1] Review of Modern Physics **81**, 592 (2009).
- [2] Journal of Statistical Mechanics **P08029** (2007).
- [3] Journal of Physics A : Mathematical and General **37**, 8479 (2004).

Nature of the project : ~70 % analytical (master equation, generalized diffusion equation, differential equations) and ~30 % numerical (solving ODEs and stochastic simulations).

Keywords : stochastic and nonlinear dynamics, individual-based modelling, complex systems.

Pre-requisites :

- MATH2750 (Introduction to Markov processes)
- MATH1920 (Computational mathematics)
- MATH2391 (Nonlinear differential equations)

Supervision: This project and lines of investigation have been proposed and will be supervised by Dr Mauro Mobilia (Email: M.Mobilia@leeds.ac.uk, phone: 31591).

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Project suitable for undergraduate and summer bursary students

Prerequisites: for this project it is required and assumed that the students have a

- **Good knowledge of calculus and linear algebra**
- **MATH1920 : Good knowledge of scientific computing (Mathematica, Matlab, ...)**
- **MATH2391 : Good knowledge of the theory of nonlinear differential equations**
- **MATH2750: Good knowledge of probability and stochastic (Markov) processes.**

The student's commitments:

- According to the module catalogue: **142 hours of private study** for MATH3422/MATH3423, and respectively **287 and 383 hours** of private study for MATH5003 and MATH5004.
- To punctually attend all meetings, being duly prepared, with the supervisor
- To swiftly inform the supervisor if s/he cannot attend a meeting and/or of any changes concerning the previously agreed work-plan
- To work regularly by himself/herself on the project, as instructed by the supervisor.
- To search the literature and do the relevant reading
- To write an independent maths report on the project
- For *summer bursary students*: A written report will have to be submitted at the end of the project and the student will give an oral presentation. Please refer to the scheme's description for more details.

The supervisor's commitments:

- According to the module catalogue: **8 hours** of supervision meetings for MATH3422/MATH3423, and respectively **13 and 16 hours** of supervision for MATH5003 and MATH5004.
- To provide scientific guidance, e.g. concerning tasks and relevant literature
- To punctually attend all meetings
- To swiftly inform the student if he cannot attend a meeting and/or of any changes in what had been previously planned

What the student can expect from the supervisor:

- To provide scientific guidance, e.g. concerning tasks and relevant literature
- Regular supervision meetings
- Scientific guidance
- Reference to the appropriate literature
- Fair assessment and appropriate feedback

What the student cannot expect from the supervisor:

- The supervisor will not write any parts of the student's report
- The supervisor will not perform any calculations for the student
- The supervisor will not search the literature, order books or articles for the student.

Assessment: see the module catalogue. *There will be an oral presentation of the project (oral assessment or viva) and a discussion of the report and of its presentation. The main assessment criterion concerns the mathematical understanding and ability demonstrated in the report and in its oral presentation.* The assessment will take into account how much of the original project was eventually covered, and the understanding demonstrated by the student.

