MATH 3567/MATH 5567M: (Advanced) Evolutionary Modelling

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Office hour: I will be in my office and available for questions/discussions on the course and/or on the examples from 15:00–16:00 on Tuesdays, during teaching weeks.

MATH 3567/MATH 5567M are co-taught level 3/5 mathematics modules.
"MATH 3567: Evolutionary Modelling" is a 15-credit module delivered in 3 weekly lectures, and "MATH 5567M: Advanced Evolutionary Modelling" is a 20-credit module delivered in 4 weekly lectures.

- **Lectures:**
  - All **Mondays** of Weeks 1-11: joint MATH3567/5567M lectures will take place from **10am - 11am** in **Roger Stevens LT 10**
  - All Tuesdays of Weeks 1-11: joint MATH3567/5567M lectures will take place **from 9am - 10am** in **Roger Stevens LT 11**
  - All **Fridays** of Weeks 1-11: joint MATH3567/5567M lectures will take place from **9am - 10am** in **Roger Stevens LT 11**
  - All **Fridays** of Weeks 1-10: stand-alone MATH5567M lecture will take place from **10am - 11am** in **Roger Stevens LT 11**
  - *Special arrangements in Week 11*: Lectures on Monday 04/05 and Tuesday 05/05 are as normal. In addition, lectures on Wednesday 6th May 2020 as follows: **On Wed. 06/05 there will be a joint MATH3567/5567M lecture from 10:00-11:00 in Roger Stevens LT 18, followed by a stand-alone MATH5567M lecture from 11:00-12:00 also in Roger Stevens LT 18.** *There will be no lectures on Friday 8th May 2020 (University will be closed for "Early May Bank Holiday")*

  Full lecture notes will regularly be put on Minerva, but paper copies of the lecture notes will *not* be distributed. These notes will be posted on Minerva in due course and cover the material common to MATH3567 and MATH5567M. The notes also comprise well-recognizable sections dedicated to some extra topics that will be covered on Fridays (10:05-10:55) during the stand-alone MATH5567M lectures. During lectures, this material will be presented and explained. In addition, the lecture notes will contain a small number of well-recognizable *optional sections* that contain *non-examinable* material on some more advanced topics. You will probably want to complete the lecture notes with your own hand-written notes and comments based on what will be written on the board. (It may not be necessary to write everything since the material is also in the online notes.) After each lecture you are encouraged to consult the online material and your own notes.

  Please note that the MATH3567 and MATH5567M learning resources (lecture notes, example sheets, solutions, lecture captures, etc), info, corrections, and announcements **will be posted on the single Minerva space coded MM9104**. *You are hence advised to regularly check the course updates on Minerva under MM9104.*

  If you miss one or more lectures, it will be your responsibility to catch up with the material covered in the lectures that you missed.

- **Revision Session**

  There will be an optional, yet timetabled, joint MATH3567/5567M revision session on **Thursday 14th May** from **10:00-11:00** in the **Chemistry West Block LT F (G.74)**.

- **Example Sheets**

  The lectures will be completed by a series of 5 example sheets (one every other week, from Week 2 onwards) that will help you revise and integrate the concepts seen in the lectures. Example sheets and their solutions will be posted on Minerva. Each example sheet will consist of a number of questions that are designed for MATH3567/MATH5567M students and other questions (identified by an asterisk) that are only for MATH5567M students. You will need to work out the technical details by yourselves that is part of the learning process for which private study is expected (see below). Detailed solutions of the example sheets will be posted regularly on Minerva, and one question per sheet will be a worked out example discussed in the lecture notes. *Example sheets are an integral component of the MATH3567/5567M modules.*

  Other relevant info:

  - Example sheets **are not marked** and do not contribute to the final mark.
  - Yet, as example sheets and all examples discuss during the classes are **integral parts** of the modules, their content **is examinable**
  - Detailed solutions of all example sheets will be posted on Minerva in due course.
  - You are very much encouraged to submit your solutions to the example sheet questions to the course teaching assistant who is **Leonardo Miele**: If you submit your solutions in Leonardo's pigeon-hole at level 8 of the School of Maths (down the stairs from the Reception) by the deadline given in the chart below, he will provide you with useful written feedback *(but no marks)* on your work. Commented work will tentatively be returned within one week from the submission deadline, and will be put into a dedicated blue tray at level 8 of the School of Maths where it will be available to be collected. There will be no feedback on work submitted late.

  The example sheets are organised as follows:

Example Sheets	*Posted on*	*Deadline to submit work to Leonardo*	*Solutions posted on*
Example Sheet 1	Tu. 04/02	Thu. 13/02 by 12 noon	Fri. 14/02
Example Sheet 2	Tu. 18/02	Thu. 27/02 by 12 noon	Fri. 28/02
Example Sheet 3	Tu. 03/03	Thu. 12/03 by 12 noon	Fri. 13/03
Example Sheet 4	Tu. 17/03	Thu. 26/03 by 12 noon	Fri. 27/03
Example Sheet 5	Tu. 28/04	Wed. 06/05 by 12 noon	Wed. 06/05

Assessment

The assessment of MATH3567 and MATH5567M is entirely based on a written exam (one for each module). There will be 4 questions to answer (out of 5), each of which will carry the same marks.

- For MATH3567: there will be a 2.5 hour exam in May/June, accounting for 100% of the final grade.
- For MATH5567M: there will be a 3.0 hour exam in May/June, accounting for 100% of the final grade.

Private study

Private study is an integral part of the learning process for MATH3567 and MATH5567M. According to the "Module and Programme Catalogue", MATH3567 and MATH5567M students are respectively expected to dedicate 10.6 and 14.2 hours/week to *study and revise the course material, read as directed, and to complete*

the example sheets. For this, the weekly lecture notes are posted ahead of the first class of each week, and an example sheet will be posted every two weeks. You will then have 10 days to work on an example sheet by yourself (with support from the lecture notes and worked out examples), and will have the opportunity to receive written feedback from the course teaching assistant (see above).

Course information

Information (handouts, announcements, example sheets, papers, solutions, corrections) will be added regularly to the course page at the Minerva Blackboard accessible from: `https://minerva.leeds.ac.uk/`

Objectives of MATH3567/MATH5567M

Darwin's natural selection paradigm is a cornerstone of modern evolutionary biology and ecology. Darwinian ideas have applications in social and behavioural science, and have also inspired research in the mathematical and physical sciences. In the last decades, mathematical analysis and theoretical modelling have led to tremendous progress in the quantitative understanding of evolutionary phenomena. Yet, many questions of paramount importance, like the "origin of cooperative behaviour" or "what determines biodiversity", are subjects of intense research and their investigation requires advanced mathematical and computational tools.

The students of the co-taught MATH3567/MATH5567M modules will be exposed to fundamental ideas of evolutionary modelling. These will be introduced through influential models and paradigmatic examples that will be analysed by a combination of methods drawn from the theory of nonlinear dynamics and stochastic processes. The students of these modules will thus be introduced to some areas of applied mathematics that currently give rise to exciting new developments and prominent challenges in mathematical biology and in evolutionary dynamics.

Provisional Schedule

1. Introduction to evolutionary modelling
2. Modelling with difference equations
3. Modelling with ordinary differential equations
4. Introduction to Mendelian genetics
5. Introduction to game theory
6. Evolutionary game theory
7. Random processes: Discrete-time Markov chains
8. Random processes: Continuous-time Markov chains & birth-and-death processes
9. Evolutionary games in finite populations
10. Diffusion theory, Fokker-Planck equations & applications
11. Application of diffusion processes to population genetics

Reading list & References

The lecture notes and those that you will take in class will be sufficient to complete the example sheets and to revise for the exams. However, you may find it helpful to use the following (main) references for further details and/or to practice questions, all of which are available at the Library (links provided on Minverva/VLE):

- L. J. S. Allen, *An Introduction to Mathematical Biology* (Pearson, 2007).
- S. H. Strogatz *Nonlinear Dynamics and Chaos* (Westview Press, Cambridge, MA, 2000).
- N. F. Britton, *Essential Mathematical Biology* (Springer-Verlag, London, 2003).
- G. de Vries, T. Hillen, M. Lewis, J. Müller, B. Schönfisch, *A Course in Mathematical Biology* (SIAM Society for Industrial and Applied Mathematics, Philadelphia, 2006).
- J. Maynard Smith, *Evolutionary Genetics* (Oxford University Press, Oxford, 1998).
- L. J. S. Allen, *An Introduction to Stochastic Processes with Applications to Biology* (Upper Saddle River, N.J.: Pearson Prentice Hall, 2003).
- M. A. Nowak, *Evolutionary Dynamics* (Belknap Press, Cambridge, MA, 2006).
- M. Broom and J. Rychtář, *Game-Theoretical Models in Biology* (CRC Press, Boca Raton, USA, 2013).

Specific references (books and chapters) will be given for each topic.