

# MATH5835M Statistical Computing

## Exercise Sheet 4

<http://www1.maths.leeds.ac.uk/~voss/2018/MATH5835M/>

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*This does not count towards your final mark, the questions are for self-study only. We will discuss the answers to these questions in the lecture on 6th December.*

**Exercise 13.** Let  $X \sim \mathcal{N}(0, 1)$ .

- a) Show that  $\mathbb{E}(1 - X^2/2) = 1/2$ .
- b) Explain how the control variates method, together with the result from part a, can be used to compute an estimate for the expectation  $\mathbb{E}(\cos(X))$ .

**Exercise 14.** Let  $a < b$  and  $0 < c < d$  be given constants, and  $f: [a, b] \rightarrow [c, d]$  be a given function. Consider the following rejection algorithm:

- i Generate  $(X, Y)$  uniformly distributed on the rectangle  $[a, b] \times [c, d]$ .
- ii If  $f(X) < Y$ , output  $X$
- iii Go back to step i.

What is the distribution of the numbers which this algorithm outputs?

**Exercise 15.** Let  $(X_n)_{n \in \mathbb{N}_0}$  be a sequence of independent,  $\mathcal{N}(0, 1)$ -distributed random variables. We can consider this sequence to be a Markov chain. What is the transition density and the density of the initial distribution for this Markov chain?

**Exercise 16.** Let  $(x_1, \dots, x_m) = (3, 1, 4, 1, 5, 2)$  and let  $Y$  be one of the  $x_i$ , chosen uniformly at random, *i.e.*  $Y = x_I$  where  $I \sim \mathcal{U}\{1, 2, \dots, m\}$ .

- a) What is  $P(Y = 1)$ ?
- b) What is  $\mathbb{E}(Y)$ ?
- c) What is  $\text{Var}(Y)$ ?