

MATH5835M Statistical Computing

Exercise Sheet 2

<http://www1.maths.leeds.ac.uk/~voss/2018/MATH5835M/>

Jochen Voss, J.Voss@leeds.ac.uk

2018/19, semester 1

This does not count towards your final mark, the questions are for self-study only. We will discuss the answers to these questions in the lecture on 25th October.

Exercise 5. Let $X \in \mathbb{R}$ be a random variable and $f, g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions. Consider the control variates estimator

$$Z_N^{\text{CV}} = \frac{1}{N} \sum_{j=1}^N (f(X_j) - g(X_j)) + \mathbb{E}(g(X))$$

for $\mathbb{E}(f(X))$. Give a proof that Z_N^{CV} is unbiased and has mean squared error $\text{MSE}(Z_N^{\text{CV}}) = \text{Var}(f(X) - g(X))/N$.

Exercise 6. Let $x_0 = 0$ and $x_n = \cos(x_{n-1})$ for all $n \in \mathbb{N}$. Use R to compute x_{20} .

Exercise 7. Let $U \sim \mathcal{U}[0, 1]$ and $X = 1/U^{3/2}$. Create 1000 samples from the distribution of X and create a *meaningful* histogram of your samples.

Exercise 8. The following function is a (failed) attempt to compute

$$\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2,$$

i.e. the sum of squared increments, in R:

```
SomethingWrong <- function(x) {  
  n <- length(x)  
  sum <- 0  
  for (i in 1:n-1) {  
    sum <- sum + (x[i+1] - x[i])^2  
  }  
  return(sum)  
}
```

When we apply this function to the vector (1,2,3), we do not get the correct answer 2, but `numeric(0)` instead.

```
> SomethingWrong(c(1,2,3))  
numeric(0)
```

What is the mistake in the function `SomethingWrong`?