MATH5835M Statistical Computing
Exercise Sheet 1

`http://www1.maths.leeds.ac.uk/~voss/2018/MATH5835M/`
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This does not count towards your final mark, the questions are for self-study only. We will discuss the answers to these questions in the lecture on 11th October.

Exercise 1. Let $U \sim \mathcal{U}[0,1]$ and $X = U^3$.

a) Compute the exact value of $\mathbb{E}(X)$ analytically.

b) Compute an estimate for $\mathbb{E}(X)$ using Monte Carlo estimation.

Exercise 2. Let $\hat{\theta} = \hat{\theta}(X)$ be an estimator for $\theta \in \mathbb{R}$. Give a proof that

$$\mathrm{MSE}(\hat{\theta}) = \mathrm{Var}(\hat{\theta}) + \mathrm{bias}(\hat{\theta})^2.$$

Exercise 3. Let $X \sim \mathcal{N}(0,1)$ and consider estimating the probability $p = P\big(\sin(X) > 1/2\big)$ using a Monte Carlo estimate. How many Monte Carlo samples are required to get an estimate for p which is correct to n decimal places, for $n = 1, 2, 3, 4, 5, 6$? How long would it take to compute these estimates on your computer?

Exercise 4. Let $X, Y \sim \mathcal{N}(0,1)$. Compute a Monte Carlo estimate for $\mathbb{E}\big(\max(X,Y)\big)$ which has a relative error of less than 1%. (Hint: The R functions `pmax()` or `ifelse()` can be used to compute the maximum of X_j and Y_j.)