Some MSc/PhD topics in probability by Prof A.Veretennikov (after Intro Markov processes MATH2750)

1. The question of *existence of a general continuous time Markov process*[1] with countable state space with given transition intensities has only a partial answer to this day. (The problem does not show up in finite state spaces.) There are reasonable sufficient conditions that do not allow a rapid growth of intensities at $+\infty$, which are sufficient for existence and uniqueness. My expectation is that in the next one or two decades this topic will be attacked by a new generation of scientists. The problem is of great interest for various applications. Cf. [S. Karlin and H. M. Taylor, A First Course in Stochastic Processes, 2nd Ed., New York et al., Acad. Press, 1975].

2. In 2009 there was a celebration of the paper 1909 by *Erlang* that gave rise to a big branch of applied mathematics, queueing theory. Erlang's formulae state stationary regime of the Markov birth-death process, but not rate of convergence to that regime. Conditions that guarantee exponentially fast convergence are known for about 50 years; yet, they could be probably extended and generalized. Also, yet there is no general study with classification so as to say in which case we may have a slower polynomially fast or a sub-exponentially fast convergence. Cf. [On mixing rate and convergence to stationary regime in discrete time Erlang problem, Automation and Remote Control, 70(12), 2009, 1992-2002] and [On mixing rate and convergence to a stationary distribution in continuous-time Erlang-type systems, Problems of Information Transmission, 46(4), 382-389], with current links[2] http://www.springerlink.com/content/yh7u882508n31671/ and http://www.springerlink.com/content/15q357836600w953/, but yet many important questions in this area are still open, for example, *Estimating rate of convergence depending on the moments of initial distribution*[3], *Rate of convergence under weak recurrence conditions*[4], *Scaling and convergence rates* (see also section 4 below)[5], etc.

[1] PhD rather than MSc

[2] Valid as on 07.03.2011

[3] MSc and PhD

[4] PhD rather than MSc

[5] MSc and PhD

3. More general, *non-exponential service time* and the problem of stationary – in the sense that should be accurately defined – distribution was considered in 50s by R. Fortet (1956) and B. Sevastyanov (1957), later by some others including your lecturer (1977), (1997). In the last of those papers [M. Kelbert and A. Veretennikov, On the estimation of mixing coefficients for a multiphase service system, QUESTA, 25(1-4), 1997, 325-337], some case with exponentially fast convergence to "stationary regime" was studied. However, those results possibly may be extended, and there are still no general results about slower convergence rates, important in many applications; hence, there are good opportunities to advance in this highly important area.[6]

4. *Rescaling and extinction problems* for birth-death processes describing populations and asymptotical formulae for finial extinction are of great importance in biology, chemistry, etc. Rescaling means recognition that population normally lives at "large states" $X_t >> 1$, at some level, so a reasonable variable could be some rescaling of X, i.e. $Y_t = X_t/N$ for certain $N >> 1$; nevertheless, despite the fact that normally the value of the process is very large, extinction could be possible, and to find its asymptotics is a very important applied problem. Here is a useful link to the preprint submitted in 2009, [M. Mobilia, M. Assaf, Fixation in Evolutionary Games under Non-Vanishing Selection, http://arxiv.org/abs/0912.0157]. There are great open problems in this direction, e.g., *Large and moderate deviation asymptotics of extinction*[7],

[6]PhD rather than MSc
[7]MSc and PhD