

MATH3733, PLAN OF THE COURSE

This is the plan of the course. Roughly, you are going to study **two major parts**: discrete time and continuous time theory of option pricing. The discrete time part is easier; however, the main goal of the course is to learn continuous time counterpart, in particular, the **Black-Scholes formula**.

This is a mathematics, thus, it is rigorous. Hence, the first advice is: **learn exact definitions** of all notions used in the theory. Another advice is to learn definitions of all basic notions you already know from previous years: it will be **not** sufficient to know them intuitively for this course. In the very beginning there will be a test which will help you realize what is needed.

To help you follow the whole course, here is its (approximate) plan below.

- European options, introduction. The main question is: how to value a European option? You will learn the notion of arbitrage, one of the main tools of the theory. Lectures 1 – 4.
- Reminder of basic probability notions, and some new theoretical material: lectures 5 – 6. You will learn new terms such as conditional expectation, martingale and discounted martingale.
- Simplest “binomial” model, option pricing for this model: lectures 6 – 9. You will also learn that option price is a (discounted) martingale after a strange transformation: change of probabilities; new probabilities are called implied ones, a very important notion. How better acquire the concept of change of probabilities: read the first two pages in the book [BR] “The parable of the bookmaker” which explains that the latter values the bets not following real chances (probabilities) of each horse to win, but rather following the bets themselves, – this is the change of probabilities in practice.
- Reminder of the Central Limit Theorem (which you know from previous courses); application of this theorem to the first appearance of the Black-Scholes formula, the goal of the course – lectures 10 – 13.
- Elements of stochastic calculus: this is a new material and it will appear difficult. I will try to explain everything at the intuitive level. However, there is no way to learn this part other than real hard study, like ancient Greeks told that “There is no royal way to mathematics”: **you** must learn it. New notions you will learn: Brownian motion, stochastic integral, stochastic differential equation, Cameron - Martin - Girsanov transformation of measure. Lectures 14 – 20.
- Continuous time model of stock price; Black-Scholes formula via solving the basic Black-Scholes partial differential equation and via transformation of measure. Lectures 21 – 24.
- Two lectures for revision, lectures 25 – 26. Other slots - for example classes.