1 MATH3733

1.1 Introduction

The modern theory of stochastic financial mathematics was created during last 25-30 years, though started in the very beginning of 20th century by L. Bachelier, see http://cepa.newschool.edu/het/profiles/bachelier.htm. Two of the founders of the modern theory received a Nobel prize in economics in 1997: Robert S. Merton (http://almaz.com/nobel/economics/1997a.html) and Myron S. Scholes (http://almaz.com/nobel/economics/1997b.html). One more Nobel prize winner related to stochastic finance is Paul A. Samuelson (http://www.almaz.com/nobel/economics/1970a.html). The theory is indeed used in financial world.

Knowledge required for a person who wishes to become a financial analyst:

1 economics background;

2 stochastic analysis (Wiener process, Itô's formula, martingales, Girsanov's transformation);

3 approximation methods of solving partial differential equations (PDEs) and stochastic differential equations (SDEs);

4 computer modelling (including computer language skills).

You will study in this module mainly part 2, and only a bit of 1, 3 and 4 (the latter on your practical). Computer language used is **R**. one may find helpful resources on the School Web pages.

Elementary economics background, some notions

1.2 Markets

- **Stock markets.** *Stock* is a printed paper issued by a company to attract investments, money. Usually with dividends. [NYSE (New York Stock Exchange), NASDAQ (National Association of Securities Dealers Automatic Quotation), EUSDAQ, London Stock Exchange, et al.]

- **Bond markets.** "*Bond*: a printed paper given by a government, city or business company saying that money has been received and and will be paid back, usually with interest," which is fixed.

- **Currency markets or foreign exchange markets.**
- **Commodity markets** (oil, gold, copper, wheat, electricity, etc).
- **Futures and options markets.** "*Futures*: goods and stocks bought at prices agreed upon at the time of the purchase but paid for and delivered afterwards." "*Option*: the right to buy or sell something at a certain price within a certain period of time." [CBOE (Chicago Board Option Exchange).]

1.3 Why study option models and option pricing?

Because there is a huge financial industry which requires a corresponding mathematics. Options is just a simplest example.

1.4 What is option

A *European call option* is a *contract* with the following conditions ([WHD, section 1.2]):

- At a prescribed time in the future, known as the expiry date or expiration date or simply expiry T, the holder of the option *may*
- purchase a prescribed asset, known as an *underlying asset* or, briefly, the *underlying*, for a
- prescribed amount, known as the *exercise price* or *strike price*, K.
- The call option can be purchased for a price C_t called the *premium* at time t, $0 \leq t < T$. The price varies with t.

Emphasize, it is a *right* and not an *obligation* for the holder of this contract. The other party of the contract who is known as the *writer*, does have a potential obligation: he **must** sell the asset if the holder chooses to buy it.

The problem of option pricing is the problem how determine the right premium.

A *European put option* is similar, but gives the owner the right to *sell* an asset at a specified price ar expiration.

In contrast to European options, *American options* can be exercised *any time* between the writing and the expiration of the contract. Both types of options are traded all over the world, independently of their titles, as well Asian, Russian, etc.

We always suppose that there is *no commissions or fees, nor dividends*, though all this in principle may be considered in more advanced theories.