Exercises 4

1. Suppose XYZ stock is selling for \$115 per share in March and an XYZ September 100 call is selling for \$30. Assuming European options and an interest rate of 6% per year, what is the value of the XYZ September 100 put?

2. Use arbitrage arguments to confirm the following inequalities on European call options, where S is the current stock price, t is the current time, T is the expiry time, r is the interest rate, and k is the exercise price.

(a) $C \leq S$.

(b) $C \geq S - ke^{-r(T-t)}$.

(c) If two otherwise identical calls have exercise prices k_1 and k_2 with $k_1 < k_2$, then

$$0 \leq C(S, t; k_1) - C(S, t; k_2).$$

(d) In the same setting,

$$C(S, t; k_1) - C(S, t; k_2) \leq k_2 - k_1.$$

Can you strengthen this last inequality?

3. Consider a two-step binomial tree model with structure described below. The interest rate r for one time step satisfies $e^r = 4/3$. Find the value at time $t = 0$ of an option which pays £100 if the final stock value exceeds its initial value. Stock values are given in pounds. Stock prices are:

$S_0 = 30$; S_0^+ (jump up from S_0) $= S_1^1 = 60$, S_0^- (jump down from S_0) $= S_1^2 = 30$;

$(S_1^1)^+$ (jump up from S_1^1) $= S_2^1 = 110$, $(S_1^1)^- = (S_1^2)^+ = S_2^2 = 50$,

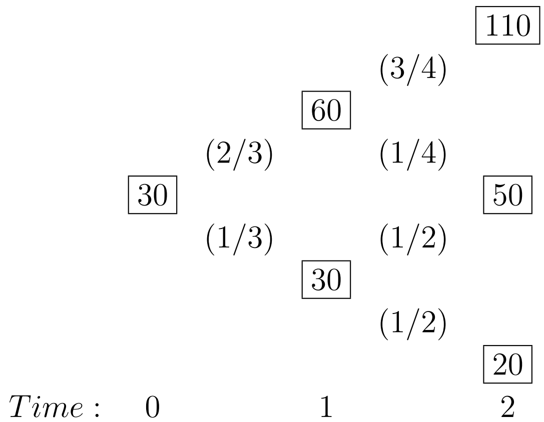
$(S_1^2)^-$ (jump down from S_1^2) $= S_2^3 = 20$,

and probabilities

p_0(jump up) $= 2/3$, p_1(jump up from S_0^+) $= 3/4$, p_1(jump up from S_0^-) $= 1/2$.

[**Continued on the next page...**]

It is recommended to use the following graphical representation of the question, and also an advice is to keep the structure of this picture while you are computing implied probabilities, etc.



4*. Compute the stochastic differential of the process $(1+W_t^4)e^{W_t}$. Hence or otherwise, prove that $E(1+W_t^4)e^{W_t} > 1$ for any $t > 0$.

5**. Solve an SDE $dX_t = X_t dW_t + dt,\ X_0 = 2$.

6. In a stochastic market, the share of stock price is modelled by a random process $S_t = S_0 e^{2W_t + 3t}$. Explain what is called trend and volatility for this model. Write down the Black-Scholes equation for the European put option price C_t with expiry $T = 2$ and strike price $K = 4$.

7*. Compute the value $E\left(e^{W_1^2}\right)$ (here $(W_t, t \geq 0)$ is a Wiener process).

NB: * means "more difficult, not compulsory"; ** means similar in a stronger sense.