MATH3733 EXERCISES 3

1 Assuming $\int_{-\infty}^{\infty} \frac{1}{(2\pi)^{1/2}} \exp(-x^2/2)\, dx = 1$ and $T > 0$, show

(a)

$$\exp(-rT) \int_{a/\sqrt{T}}^{\infty} b \exp(x\sqrt{T} + (r - 1/2)T) \frac{1}{(2\pi)^{1/2}} \exp(-x^2/2)\, dx = b\,\Phi\left(\frac{-a + T}{\sqrt{T}}\right),$$

and

(b)

$$\exp(-rT) \int_{a}^{\infty} b \frac{1}{(2\pi)^{1/2}} \exp(-x^2/2)\, dx = b \exp(-rT)\Phi(-a),$$

where Φ is the Laplace function,

$$\Phi(z) = (2\pi)^{-1/2} \int_{-\infty}^{z} e^{-x^2/2}\, dx.$$

2 Let

$$\boxed{C(0, S) \equiv C_T(0, S) = S\Phi(d_+) - \exp(-rT)K\Phi(d_-),}$$

where

$$d_\pm \equiv d_\pm(T) = \frac{\log(S/K) + (r \pm \sigma^2/2)T}{\sigma\sqrt{T}}, \quad \sigma > 0.$$

[**This is the Black-Scholes formula for the European call option.**]
Assuming S, $K > 0$, show

(a) *

$$C(0, S) \geq 0.$$

(b) Find the derivative

$$\Delta(0, S) \equiv \Delta_T(0, S) := \frac{\partial C_T(0, S)}{\partial S}$$

(c) and show

$$0 < \Delta(0, S) < 1.$$

(d) Calculate the limit of $\Delta_T(0, S)$ as $T \to 0$.