

A Three Dimensional Data Set of the Fracture Network of a Granite

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Abstract

A data set describing the three dimensional fracture network of a granitic rock material is presented. The scale is on the order of 1 m^3 . The purpose is to provide the scientific community with a true data set which may be used to validate models and simulations of fractured crystalline rock masses.

1. Introduction

In the modern world, the use of underground repositories for the storage of hazardous waste material is a fact and until such waste can be rendered harmless by new technology such repositories will probably increase in number despite public objection. By definition, hazardous waste presents a risk however great or small to human life, therefore, the control of the waste is of great importance. In this case, control means to determine the location of the hazardous substance over time such that it remains in the repository and does not leak.

One of the factors which determine whether a hazardous material escapes is the characteristics of the rock mass in which it is contained. The rock mass could be considered as the last line of defence to the biosphere. Thus, the ability to predict the trajectories of pollutants through the rock mass is highly desirable.

Environmental accidents are also a fact of the modern world and are well documented. In these cases, land may become contaminated with a hazardous material. Although the remediation of such land would be a priority, some of the pollutant may escape by flowing through the natural discontinuities of the ground. Again, the characteristics of the rock mass would be crucial to the manner and extent of the seepage.

There may be circumstances in which the causes of environmental pollution may not be obvious. In this case, it would be beneficial to be able to use the properties

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of the rock mass to predict the likely cause(s) from the known effect(s) and thereby prevent further contamination of the environment.

All the above considerations make the accurate characterisation of a fractured rock mass and the prediction of fluid flow through it essential. And the more numerous and densely populated civilisation becomes, and the wider range of toxic chemicals and substances it employs in the pursuit of its goals, the greater the need to predict and control the environment in order to ensure it remains benign and conducive to human and other life forms.

One way of predicting and to some extent controlling the environment is by reliable risk analysis and assessment: if the relevant dangers are assessed and quantified, it is possible to take commensurate measures against them. Risk assessment lends itself to statistical analysis since with only a finite number of data with which to make an assessment, the precision of the outcome will never approach certainty.

The risk associated with a rock mass, in terms of the stability of the rock mass and its flow properties, depends upon its constituent material and structure. Therefore rock mass characterisation is important in solving both support and fluid flow problems. Both support and flow properties are highly contingent on the fracture network within the rock, and this is where the present limitations in knowledge exist.

Current practice in identifying the joint structure of a rock mass involves taking scan line surveys or window maps of an exposure. These techniques can become sophisticated in that scan lines may be oriented in three mutually perpendicular directions to obtain an estimate of the three dimensional structure of the discontinuities. Most discontinuity surveys, however, are one or two dimensional. Thus the true three dimensional character of the rock mass is not directly approximated.

Since present day technology does not allow the exact three dimensional nature and structure of a rock mass to be measured, a statistical description of the rock mass is indicated. In this method, the behaviour of the rock mass and the flow trajectories of any fluids within it are expressed in terms of probabilities of outcomes.

Although distributions of trace length, aperture size, spacing, orientation, dip etc may be obtained from scan lines and window maps for the statistical prediction of rock mass behaviour, it would be better science to have the true three dimensional structure of the rock so that comparisons may be made with the predictions obtained from the limited data set to test their accuracy.

This is the major purpose of the data set presented here. The three dimensional fracture network has been directly measured as precisely as resources allowed. It is intended that the data may be used by the academic and industrial sectors to formulate statistical or other algorithms to better describe the flow properties through a crystalline rock mass for the purpose of improving risk analysis and assessment.

2. Three Dimensional Fracture Data Sets

A review of the literature revealed a plethora of available information describing the discontinuities of rock masses, however, the data is virtually all two dimensional and derived from scan line surveys and window maps. Only two original three dimensional studies were found.

One was published by Ledésert et al. [1] concerning a fractured, fine grained granite sample approximately 0.52 m x 0.35 m x 0.36 m (0.066 m³). The granite was from a quarry in La Peyratte, Deux-Sèvres, France. The data is presented pictorially as a series of window maps representing the strike patterns of the joints at equally spaced sections through the rock mass. By stacking the consecutive window maps, the three dimensional joint structure may be constructed.

The second three dimensional fracture network was an illustration by Gertsch [2], obtained from window mapping successive thin slices of a 0.14 m diameter core sample of gneiss. The scale of this data is on the order of 0.016 m³. The main theme of the paper was in providing a very practicable description of the methodology and technology involved in obtaining such a data set on various scales.

3. Rock Material Selection, Procurement and Preparation

A crystalline rock material was chosen for the task since this material is the preferred choice for repository purposes: these rock types are usually strong, more likely to be self-supporting and are relatively impermeable.

One of the difficulties encountered was procuring a suitable block of crystalline rock. Within a built-up country such as the UK, and due to budget and time constraints, it is not feasible to select and excavate rock samples oneself although this may be the preferred option: a supplier is needed. In practice, the suppliers of rocks on the scale of 1 m³ are stone quarries, however, the stone from these quarries is mainly for buildings, monuments, paving etc. In other words, the rock is quarried because it has few natural discontinuities which allows the customer to utilize its intact strength rather than be limited by the strength of its discontinuities. Fortunately, stone quarries have reject piles where less perfect and more typical rock samples may be obtained.

A granite sample was thus obtained from De Lank Quarry, Cornwall, UK. The quarry produces high quality building and ornamental stone but a suitable granite block was available from the waste pile. The size of the block was approximately 1.5 m x 1 m x 0.7 m, or on the scale of 1 m³. The block was sectioned into slabs using a 2 m diameter diamond tipped saw available at the quarry. The sectioned block is shown in Figure 1. There are seven large labs, stacked in order, each being approximately 75 mm thick. The width of the cutting blade was 15 mm which represents the distance between adjacent slabs.

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Figure 1. Sectioned granite sample.

The top and bottom surfaces of each slab were labelled “a” and “b” respectively and each slab numbered. The task was to window map all the top and bottom exposed surfaces, stack the strike patterns, and construct the three dimensional discontinuity network.

4. Discontinuity Mapping

In order to map the strike patterns of the joints, a co-ordinate system had to be chosen. In this case, a right-handed Cartesian system was adopted in which the z-axis represented the vertical direction, the y-axis was pseudo-north and the x-axis pseudo-east. The arrangement is illustrated in Figure 2. Thus the dip direction and dip angle are arbitrary and may be altered by suitable transformations.

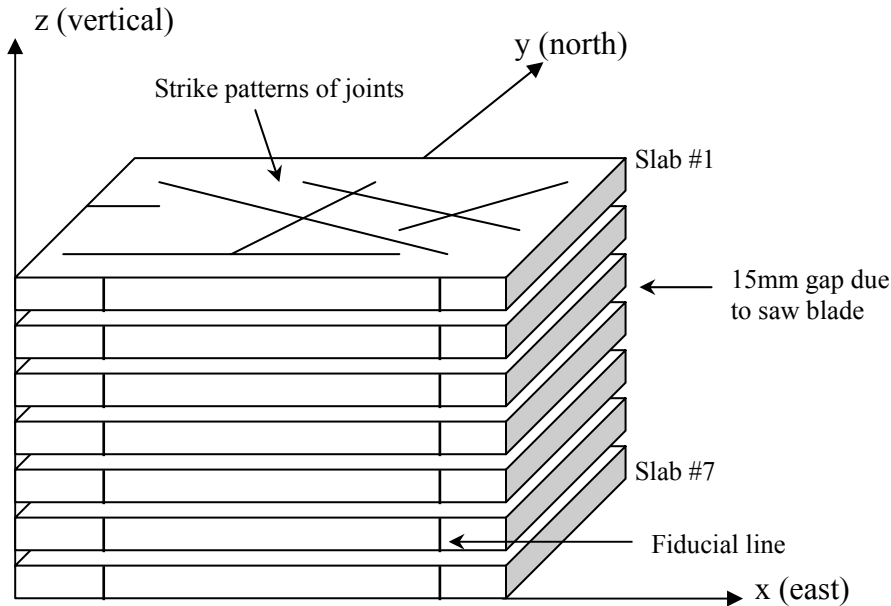


Figure 2. Idealised view of sectioned block and co-ordinate system.

Since the slabs were to fit into the same co-ordinate system, they had to be marked so they had a reference point with which to identify the common arbitrary origin. Fortunately, the chosen block had a flat side (otherwise one could have been prepared with the 2 m diameter circular saw at the quarry), which was cleaned and marked in permanent ink with fiducial lines in the z-axis direction before the block was disturbed. Subsequently, each slab was successively manoeuvred into the laboratory for window mapping. Figure 3 shows slab #2.



Figure 3. Joint measuring laboratory.

Each surface was cleaned of dust using tap water. When the water had dried some of the joints were difficult to see clearly. Therefore, a dye penetrant was applied to a smaller piece of the granite to test whether the joints would be highlighted: some were; others were obscured by the dye bringing out the structure of the large grain boundaries. Ultimately, it proved better simply to moisten the granite surface with a paint brush. This had the effect of exposing the joints without complicating the picture with the grain boundaries. A magnifying lens also proved useful for the smaller joints.

Each mapping surface was in the x-y plane but at a different height on the z-axis, Figure 2. Considering the size of the rock and for convenience, all measurements were taken in centimetres but this depends upon personal preference. The joint co-ordinates were determined using a metre rule to the nearest millimetre. Intermediate co-ordinates were recorded in addition to the terminal points of each joint, depending upon the degree of deviation from linearity of a strike line. This allowed accurate plotting of the strike patterns. Co-ordinates were also taken of the perimeter of a rock slab so that censored and uncensored joints could be identified visually from the strike patterns.

By stacking the strike patterns and following the progress of the joints through the body of the rock, the three dimensional co-ordinates of the joints were determined. For clarity, the concept is illustrated in an exaggerated manner in Figure 4. The resulting data set is called the **Leeds Rock Data Set** and comprises 392 joints.

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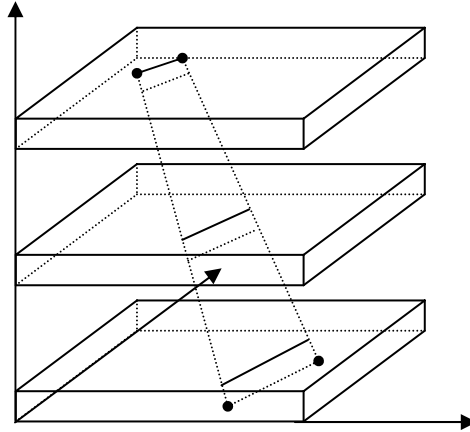


Figure 4. Joint plane obtained by stacking strike lines.

Certain joints did not show through to the other side of the same slab and therefore could not be completely defined. Time prevented further dissection or tomographic imaging of the slabs so such joints were assumed to run half the thickness of the slab. A similar assumption was made for joints that did not pass from one slab to the adjacent slab. If the strike of such a joint ran in the same direction as a joint set, it was assigned the average dip angle of that set otherwise the dip angle was assumed to be 90^0 and the joint strike was random. All these joints have been identified in the data set.

5. Joint Modelling

The immediate purpose of obtaining the three dimensional data set was to pass the data on to colleagues so they could use it for the stochastic modelling of fractured crystalline rock masses. To facilitate this, the joints were reduced to planes and ellipses in 3-space which the wider scientific community may also find useful.

5.1 Planar Representation

Most of the joints were approximately planar and those few which were curved could be subdivided into planar regions with each region treated as a separate but connected joint. Therefore, in the first instance, each joint was represented as a plane in 3-space with reference to the right-handed Cartesian co-ordinate system. The form of the equation of a plane is:

$$a.x + b.y + c.z + d = 0$$

Where x , y and z represent the co-ordinates of the plane along the orthogonal x -, y - and z -axes, and the letters a , b , c and d are constant coefficients. The coefficients were determined by fitting a plane to the corner co-ordinates of a joint using the method of least squares via a programmed spreadsheet. By this means, planes were fitted to all the joints. The geometric convention used is illustrated in Figure 5, though other conventions may be quoted in the literature. \mathbf{n} is the normal to the plane, where $\mathbf{n} = a.\mathbf{i} + b.\mathbf{j} + c.\mathbf{k}$, and \mathbf{i} , \mathbf{j} and \mathbf{k} are the unit vectors in the x -, y - and z -axis directions respectively.

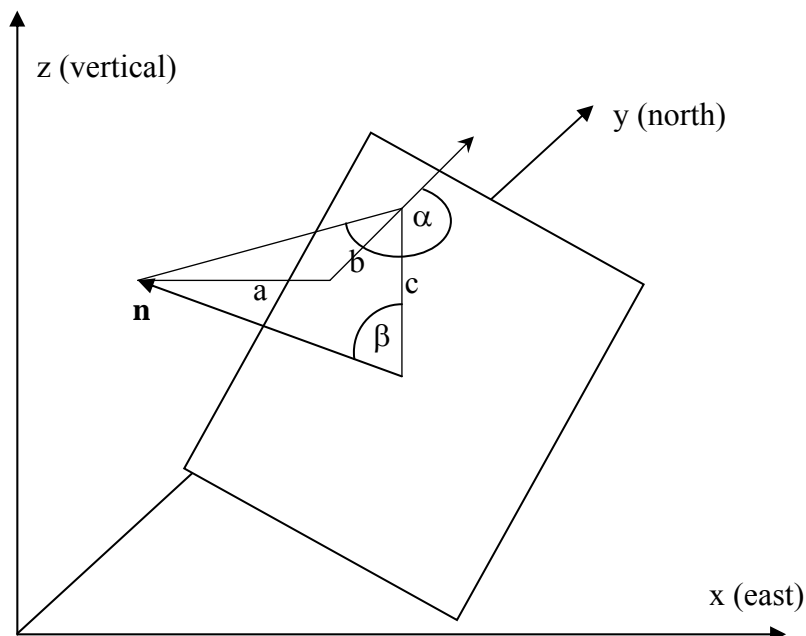


Figure 5. Geometric convention for plane representation.

From the plane equations, the dip direction and dip angle may be calculated from the following formulae:

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$$\text{Dip direction, } \alpha = \tan^{-1}\left(\frac{a}{b}\right) + Q \quad \text{where } 0^{\circ} \leq \alpha \leq 360^{\circ}$$

and Q is an angle, sometimes termed the “quadrant angle,” depending upon the sign of a and b as given in Table 1, assuming c is arranged to be positive.

a	b	Q
>=0	>=0	0°
>=0	<0	180°
<0	<0	180°
<0	>=0	360°

Table 1. Quadrant angles, Q.

$$\text{Dip angle, } \beta = \cos^{-1}\left(\frac{c}{\sqrt{a^2 + b^2 + c^2}}\right) \quad \text{where } 0^{\circ} \leq \beta \leq 90^{\circ}$$

and c is arranged to be positive. β is shown dipping from the vertical but its value is identical to the declination of the joint plane from the x-y plane as horizontal.

Because north is arbitrary in this case, the planes may be rotated in any direction relative to the original x-, y- and z-axes to change the aspect of the joint network, and new dip directions and dip angles calculated. This may be achieved by applying the following linear transformations to the components of the normals to the planes, **n**:

$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \delta & 0 & \sin \delta \\ 0 & 1 & 0 \\ -\sin \delta & 0 & \cos \delta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

where a' , b' and c' are the new components of a rotated normal. θ is the anticlockwise rotation of the x-y plane about the z-axis measured from the x-axis. δ is the anticlockwise rotation of the y-z plane about the x-axis measured from the y-axis. ϕ is the anticlockwise rotation of the x-z plane about the y-axis measured from the z-axis. The rotation angles and their positive senses are illustrated in Figure 6.

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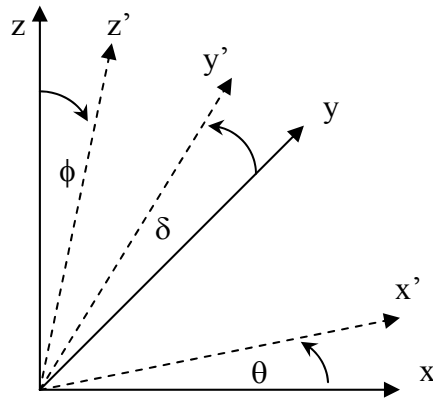


Figure 6. Rotation angles.

5.2 Elliptic Representation

Each ellipse was fitted so that it intersected the four corner co-ordinates of a joint plane in an orientation which best represented the joint. The principle is illustrated in Figure 7. Readers who undertake stochastic modelling may prefer this format.

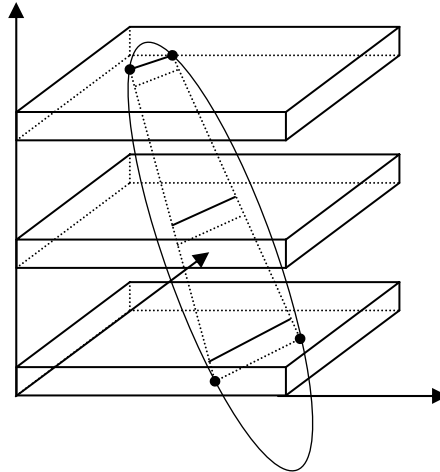


Figure 7. Ellipse fitted to a joint plane.

For the stochastic modelling undertaken at the University of Leeds, each ellipse was defined by its centre point co-ordinates, $C(x_c, y_c, z_c)$, the lengths of the major and minor axes (A_{3d} and B_{3d} respectively), the dip direction relative to the y-axis as north (α), the dip angle from the x-y plane as horizontal (β), and the angle subtended by the major axis from the dip direction in the plane of the ellipse (γ). The geometry is graphed in Figure 8.

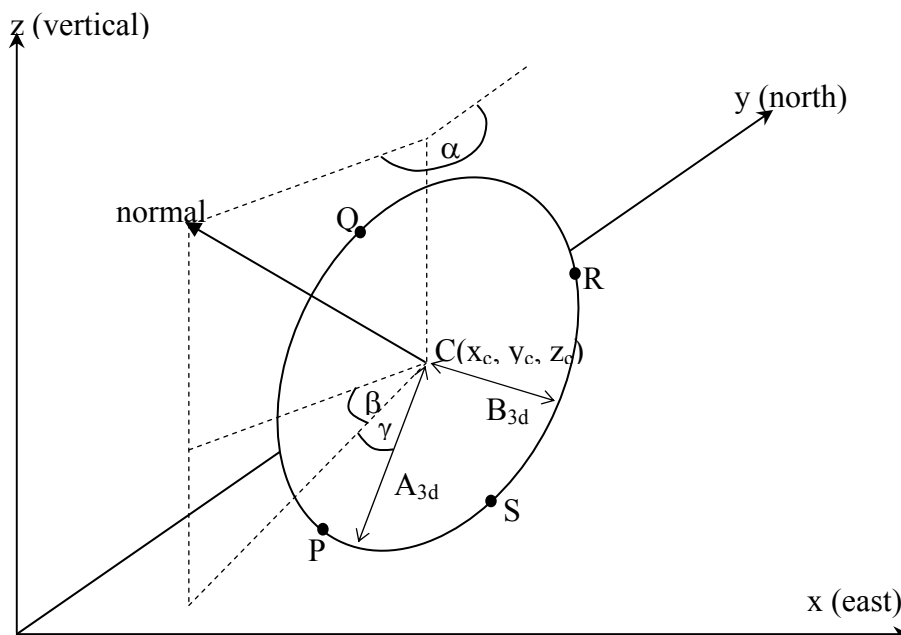


Figure 8. Three dimensional ellipse fitted to joint corner points P, Q, R and S.

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The fitting technique devised involved fitting each ellipse in two stages. Firstly, a two dimensional ellipse was fitted to the corner co-ordinates in either the x-z or y-z plane, depending upon the prevailing orientation of the joint. Secondly, this ellipse was then projected onto the previously fitted joint plane to transform it into 3-space. From the equations of the two dimensional ellipse and the joint plane, x_c , y_c , z_c , A_{3d} , B_{3d} , α , β and γ were calculated. A spreadsheet was programmed to carry out the computations. The procedure is detailed below.

The general equation of a two dimensional conic in the x-z plane is in the form:

$$a.x^2 + 2b.x.z + c.z^2 + d.x + e.z + f = 0$$

To fit an ellipse to the x-z corner co-ordinates of a joint plane, the coefficients a, b, c, d, e and f have to be determined. Since there were only four co-ordinates to which the ellipse could be fitted only four simultaneous equations could be obtained, however, there are six unknowns. Thus, the solution would have to be expressed in terms of two parameters, hence, there are an infinitely many solutions (ellipses) depending upon the arbitrary values chosen for the parameters. Therefore, the two dimensional ellipse fitting was by visual inspection using the programmed spreadsheet.

An ellipse centred about the origin with the major and minor axes oriented along the x- and z-axes respectively i.e. unrotated, has an equation of the form:

$$\frac{x^2}{A^2} + \frac{z^2}{B^2} = 1$$

Where A and B are the lengths of the major and minor axes respectively. By re-arranging, the equation may be expressed in a more standard form:

$$B^2x^2 + A^2z^2 - A^2B^2 = 0$$

In the general case where an ellipse is rotated anticlockwise from the x-axis by an angle θ , the co-ordinates suffer the following linear transformations:

$$\begin{aligned} x &\rightarrow x.\cos\theta + z.\sin\theta \\ z &\rightarrow -x.\sin\theta + z.\cos\theta \end{aligned}$$

By substitution, the general equation is obtained:

$$(A^2 \sin^2 \theta + B^2 \cos^2 \theta)x^2 + \sin 2\theta(B^2 - A^2)x.z + (B^2 \sin^2 \theta + A^2 \cos^2 \theta)z^2 - A^2B^2 = 0$$

To take into account the fact that the ellipse may be offset from the origin by the distances 'h' and 'm' in the x- and z-axis directions respectively, two non-linear transformations have to be applied:

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$$x \rightarrow x - h$$

$$z \rightarrow z - m$$

Again, by substituting these transformations into the general equation, the following is obtained:

$$\begin{aligned} & (A^2 \sin^2 \theta + B^2 \cos^2 \theta)x^2 + \sin 2\theta(B^2 - A^2)x.z + (B^2 \sin^2 \theta + A^2 \cos^2 \theta)z^2 \\ & + (k(A^2 - B^2)\sin 2\theta - 2m(A^2 \sin^2 \theta + B^2 \cos^2 \theta))x \\ & + (m(A^2 - B^2)\sin 2\theta - 2m(B^2 \sin^2 \theta + A^2 \cos^2 \theta))z \\ & + h^2(A^2 \sin^2 \theta + B^2 \cos^2 \theta) + k^2(B^2 \sin^2 \theta + A^2 \cos^2 \theta) \\ & - h.m(A^2 - B^2)\sin 2\theta - A^2B^2 = 0 \end{aligned}$$

The geometry of the transformed ellipse is illustrated in Figure 9. A and B are the lengths of the major and minor axes respectively. h and m are the centre coordinates and correspond to C_x and C_z respectively in Figure 8.

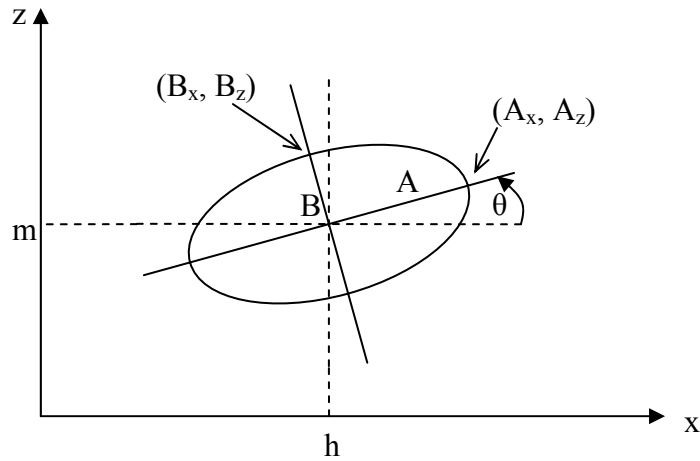


Figure 9. Geometry of transformed 2D ellipse.

By comparing the coefficients of the above equation with those of a general conic, the following relationships may be derived:

$$\begin{aligned} a &= A^2 \sin^2 \theta + B^2 \cos^2 \theta & b &= \frac{1}{2}(B^2 - A^2)\sin 2\theta \\ c &= B^2 \sin^2 \theta + A^2 \cos^2 \theta & d &= -2(ah + b.m) \\ e &= -2(b.h + c.m) & f &= a.h^2 + c.m^2 + 2b.h.m - A^2B^2 \end{aligned}$$

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By programming the above equations into a spreadsheet and inputting values for A, B, θ , h and m, an ellipse may be visually fitted to the x-z corner co-ordinates of a joint plane in the orientation which best represents the joint. The spreadsheet calculates the coefficients a, b, c, d, e and f from the formulae to give the general equation from which the ellipse is plotted for fitting purposes. An example of the programme is illustrated in Figure 10.

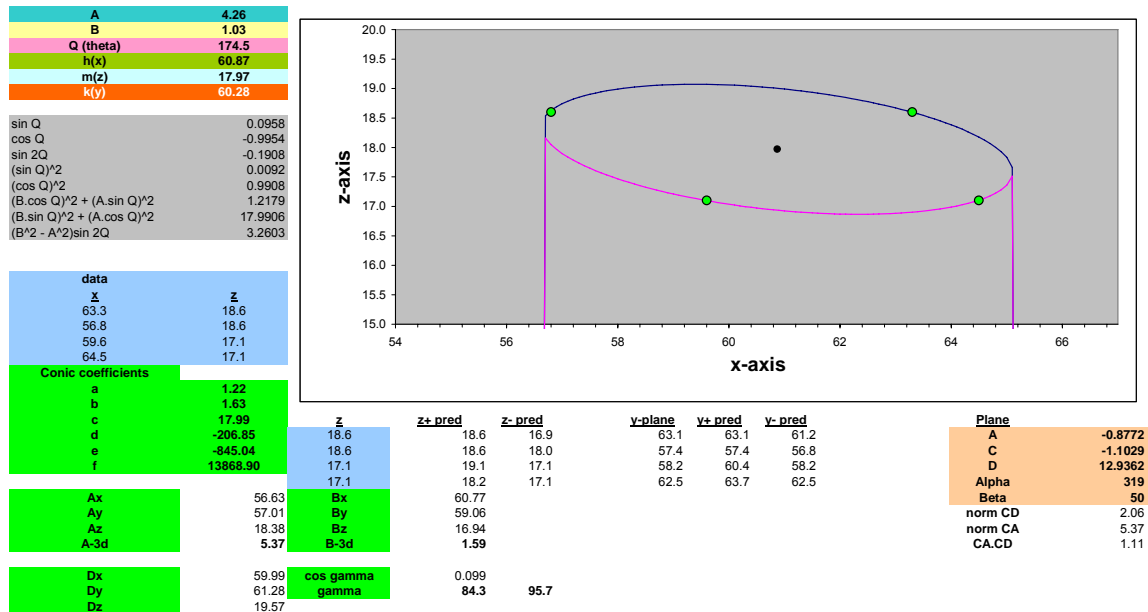


Figure 10. Spreadsheet programme for fitting two dimensional ellipses.

In Figure 10, A = 4.26 cm, B = 1.03 cm, $\theta = 174.5^\circ$, h = 60.87 cm and m = 17.97 cm. These parameters were entered by trial and error until the two dimensional ellipse fitted the four corner points of the joint in the x-z plane.

By projecting the two dimensional ellipse on to the joint plane, the ellipse is transformed into three dimensional space and the relevant properties may be calculated. For example, C_y may be obtained by re-arranging the plane equation:

$$C_y = -(a.C_x + c.C_z + d)/b$$

From Figure 9:

$$\begin{aligned} A_x &= h + A.\cos\theta &= C_x + A.\cos\theta \\ A_z &= m + A.\sin\theta &= C_z + A.\sin\theta \\ B_x &= h - B.\sin\theta &= C_x - B.\sin\theta \\ B_z &= m + B.\cos\theta &= C_z + B.\cos\theta \end{aligned}$$

And,

$$A_y = -(a.A_x + c.A_z + d)/b \quad B_y = -(a.B_x + c.B_z + d)/b$$

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Thus the lengths of the major and minor axes of the three dimensional ellipse may be calculated from:

$$A_{3d} = \sqrt{(A_x - C_x)^2 + (A_y - C_y)^2 + (A_z - C_z)^2}$$

$$B_{3d} = \sqrt{(B_x - C_x)^2 + (B_y - C_y)^2 + (B_z - C_z)^2}$$

The ellipse may be oriented in 3-space by the use of an arbitrary reference angle such as γ in Figure 8 or by expressing the centre point and the major and minor axes as vectors (written in bold case):

$$\mathbf{C} = C_x.\mathbf{i} + C_y.\mathbf{j} + C_z.\mathbf{k}$$

$$\mathbf{A}_{3d} = (A_x - C_x).\mathbf{i} + (A_y - C_y).\mathbf{j} + (A_z - C_z).\mathbf{k}$$

$$\mathbf{B}_{3d} = (B_x - C_x).\mathbf{i} + (B_y - C_y).\mathbf{j} + (B_z - C_z).\mathbf{k}$$

From these vectors, the whole circumference of the ellipse may be described by deriving the co-ordinates of a general point $D(x,y,z)$, which may be obtained from the vector \mathbf{D} , Figure 11. The vector equation is:

$$\mathbf{D} = \mathbf{C} + \mathbf{A}_{3d}\cos\phi + \mathbf{B}_{3d}\sin\phi$$

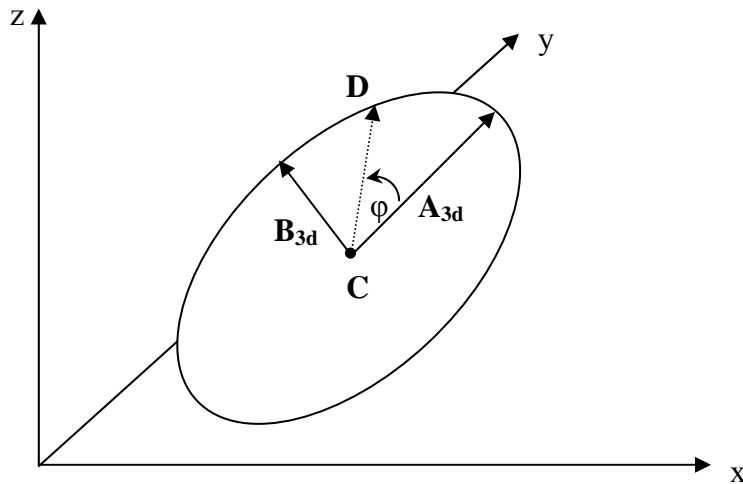


Figure 11. Vectoral representation of an ellipse in 3-space.

In practice, the fit of the ellipses to their data points were double checked in a 3D graphics plotting package previously written by a co-author for displaying simulated fracture networks. Any fine adjustments to improve the fit were made as necessary.

An elliptic representation of the entire Leeds Rock Data Set is presented in Figure 12.

The full data set may be downloaded in Cartesian co-ordinate, planar or elliptic form from the following web site: www.leeds.ac.uk/leeds/newpages/srfwww/index.htm. (Note: Proper URL to be inserted later).

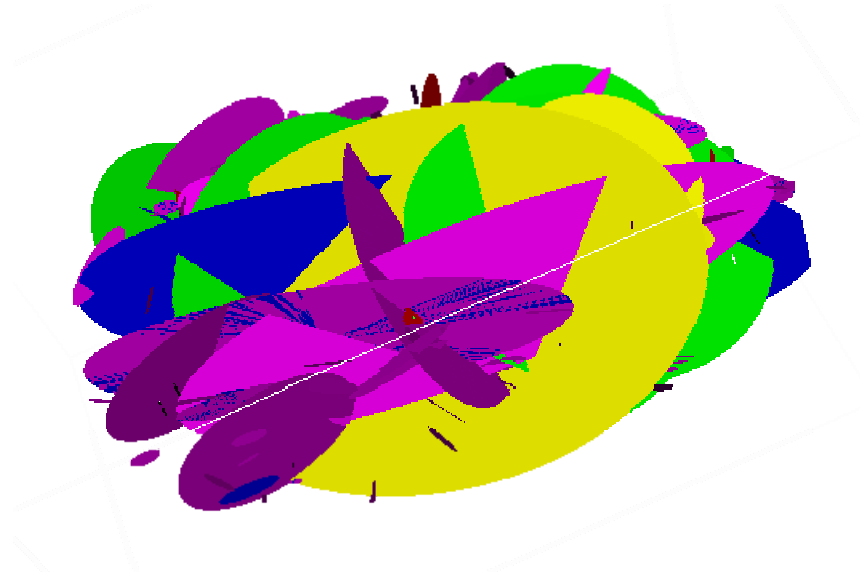


Figure 12. Elliptic representation of the Leeds Rock Data Set.

6. Conclusions

A data set, in numerical form, of the three dimensional fracture network of a crystalline rock mass on the scale of 1 m³ has been produced. The data set is suitable to be used for fracture modelling purposes such as the stochastic modelling of fractured crystalline rock masses. A literature review indicates that it is the only data set of its kind currently available to the scientific community. The data set is freely obtainable and is presented in several formats to accommodate the potential requirements of interested parties. A methodology for procuring additional data sets is described in detail.

7. Acknowledgements

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8. References

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