The Linked Twist Map Approach to Granular Mixing

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Streamline crossing structure

- Dynamical behaviour stems from intersecting streamlines
- Constant rotation rate gives analogy with blinking flows
- This can be mathematically formalised using linked twist maps
Linked Twist Maps on the plane

Domain is two intersecting annuli with two distinct regions of intersection.
A twist map takes points in an annulus...
Linked Twist Maps on the plane

... and performs a shear, wrapping this initial set around the annulus
A linked twist map is the alternate composition of such maps on a pair of annuli.

Linked Twist Maps on the plane

A linked twist map is the alternate composition of such maps on a pair of annuli.

Linked twist maps are mixing, in the sense that

$$\lim_{n \to \infty} \mu(f^n(A) \cap B) = \mu(A)\mu(B)$$

providing:

- intersections are transverse
- twists are monotonic
Blinking experiments

In the 2d tumbler, shears are monotonic, but streamlines do not cross transversely.

However the size and position of the islands can be predicted by a linked twist map analysis.
Figure 1: Mixing of initially separated blobs after 0, 5, 10 and 100 iterations for a 2-dimensional blinking tumbler.
Three dimensional blinking system
Consider a mathematical limit as the depth of flowing layer $\to 0$, and speed across flowing layer $\to \infty$.

$$F(r, \theta, x) = (r, \theta + \omega_1, x)$$
$$G(\rho, \phi, z) = (\rho, \phi + \omega_2, z)$$
$$W(r, \theta, x) = (\rho, \phi, z)$$

$$H = W^{-1} GWF(r, \theta, x)$$
Piecewise isometries can possess efficient mixing behaviour in the absence of any stretching and folding.

[see work of Arek Goetz, for example]
Comparison with non-zero flowing layer
Conclusions

- Segregation effects are crucial in studying granular flow.
- This is especially true in 2d tumbler mixers, where dynamical mixing effects are minimal.
- In 3d mixing effects can stem solely from bulk rotation in different directions.