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On the magnetically stabilizing role of the Earth's inner core

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Abstract

We consider the effect that a finitely conducting inner core may have on the dynamo processes in the outer core. Because a finitely conducting inner core has a diffusive timescale of its own of a few thousand years, which is long compared with the most rapid advective timescales possible in the outer core, the field in the inner core must necessarily average over these very rapid timescales. This averaging-out may then have a stabilizing influence, preventing the very rapid timescales from dominating the dynamo processes in the outer core. In this work we present a solution to the mean-field geodynamo equations which seems to exhibit these features. In this way it may be possible to reconcile the complicated, time-dependent nature of the field and flow in the dynamically active portion of the outer core with the simple, relatively stable nature of the externally observed dipole component of the Earth's magnetic field.

1. Introduction

The Earth's magnetic field is created in its metallic core by 'dynamo action', in which fluid motions generate a magnetic field. This metallic core consists of a solid inner core of radius $r_i =$ 1220 km, surrounded by a liquid outer core of radius $r_0 = 3480$ km. Considering that the inner core is solid, the fluid motions that generate the field are necessarily confined to the outer core, and so many dynamo models neglect the inner core entirely (Hollerbach et al., 1992), or else treat it as a non-conducting insulator (Zhang and Busse, 1990; Glatzmaier and Roberts, 1993). Here we explore the possibility that a finitely conducting inner core may nevertheless control key aspects of the dynamo process, by the following mechanism: the field in a finitely conducting inner core does not adjust instantaneously to the field in the outer core, but has a diffusive timescale of its own of a few thousand years. Once established, a field in the inner core could then have a stabilizing influence on the dynamo processes in the outer core. This could reconcile recent theoretical results (St. Pierre, 1993; Hirsching and Busse, 1994) which suggest that complicated, time-dependent flows are typically most efficient at generating magnetic fields, with the observational result that the Earth's magnetic field is in fact relatively simple and stable over long periods of time.

In two previous papers (Hollerbach and Jones, 1993a,b), hereafter referred to as papers I and II

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(see also Gubbins, 1993), we have presented a mean-field geodynamo model incorporating a finitely conducting inner core. Paper I established the basic mathematical and physical details of the model, and generally considered the new dynamics associated with a finitely conducting inner core, in particular the existence of a second Taylor's constraint requiring the integrated Lorentz torque on the inner core to vanish in the limit of vanishing viscosity. Paper II included a kinematically prescribed buoyancy force, and focused on a particularly interesting new type of solution that emerged, in which the dynamo action was concentrated in the region outside the inner core tangent cylinder. Despite the existence of very large, occasionally very rapid fluctuations in this region, the quiescent region inside the inner core tangent cylinder stabilized the external dipole component sufficiently to prevent it from reversing with every fluctuation. In this work we examine this solution in more detail, focusing on why the active region exhibits such rapid fluctuations, and why the quiescent region does not participate in them.

2. Equations

The scaled mean-field induction equations are

$$\frac{\partial \boldsymbol{B}_i}{\partial t} = \nabla^2 \boldsymbol{B}_i \tag{1}$$

in the inner core, and

$$\frac{\partial \boldsymbol{B}_0}{\partial t} = \nabla^2 \boldsymbol{B}_0 + \nabla \times (\alpha \boldsymbol{B}_0) + \nabla \times (\boldsymbol{U} \times \boldsymbol{B}_0) \tag{2}$$

in the outer core. The conductivity is thus the same in the inner and outer cores. The momentum equation in the outer core is

$$2\hat{\boldsymbol{k}} \times \boldsymbol{U} = -\nabla p + \epsilon \nabla^2 \boldsymbol{U} + (\nabla \times \boldsymbol{B}_0) \times \boldsymbol{B}_0 + \Theta \boldsymbol{r} \tag{3}$$

Here $\boldsymbol{B}$ is the large-scale axisymmetric magnetic field, $\boldsymbol{U}$ is the large-scale axisymmetric fluid flow, and p is the pressure. As in paper II, $\Theta = -\Theta_0 r \cos^2\theta$ is the kinematically prescribed buoyancy force, chosen to drive a thermal wind essentially independent of the colatitude θ. In paper I, α, which is a parameterization of the small-scale non-axisymmetric flow, was taken to be the scalar $\alpha_0 \cos\theta$; in paper II and here we retain the same spatial dependence, but take it to be a tensor [1], and include only that component that regenerates poloidal from toroidal field. The toroidal field is thus not regenerated from the poloidal field by the α-effect, but by the above thermal wind via the so-called ω-effect. Finally, the Ekman number ϵ is a measure of viscous to Coriolis forces; its value is not accurately known in the Earth's core, but it is certainly very small, much smaller than numerical considerations allow us to attain in this work. In paper II we took it to be 10^{-3}; here we have done a further run at 5×10^{-4}. This reduction by a factor of 2 thus represents a (small) step in the right direction, and allows us to estimate at least qualitatively how the resulting solution scales with ϵ.

Decomposing as

$$\boldsymbol{B} = \nabla \times (A\hat{\boldsymbol{e}}_\phi) + B\hat{\boldsymbol{e}}_\phi \tag{4a}$$

$$\boldsymbol{U} = \nabla \times (\psi\hat{\boldsymbol{e}}_\phi) + v\hat{\boldsymbol{e}}_\phi \tag{4b}$$

as in paper I, the induction equations become

$$\frac{\partial A_i}{\partial t} = D^2 A_i \tag{5a}$$

$$\frac{\partial B_i}{\partial t} = D^2 B_i \tag{5b}$$

and

$$\frac{\partial A_0}{\partial t} = D^2 A_0 + \alpha B_0 + N(\psi, A_0) \tag{6a}$$

$$\frac{\partial B_0}{\partial t} = D^2 B_0 + M(v, A_0) - M(B_0, \psi) \tag{6b}$$

[1] The editor has asked us to comment on the choice of α as a tensor. In general, the ith component of the vector $\alpha \boldsymbol{B}$ is $\Sigma_j \alpha_{ij} B_j$. The traditional choice of α as a scalar amounts to the specific choice $\alpha_{11} = \alpha_{22} = \alpha_{33} = \alpha$, and all other components equal to zero. We restrict that even further, and choose $\alpha_{33} = \alpha$, and all other components equal to zero.

and the momentum equation becomes

$$2\frac{\partial\psi}{\partial z} + \epsilon D^2 v = -N(B_0, A_0) \quad (7a)$$

$$2\frac{\partial v}{\partial z} - \epsilon (D^2)^2 \psi = M(B_0, B_0) + M(D^2 A_0, A_0) + \frac{\partial\Theta}{\partial\theta} \quad (7b)$$

where

$$\frac{\partial}{\partial z} = \cos\theta \frac{\partial}{\partial r} - \frac{\sin\theta}{r}\frac{\partial}{\partial\theta} \quad (8)$$

$$D^2 = \nabla^2 - (r\sin\theta)^{-2} \quad (9)$$

$$N(X, Y) = \hat{e}_\phi \cdot [\nabla\times(X\hat{e}_\phi)\times\nabla\times(Y\hat{e}_\phi)] \quad (10)$$

$$M(X, Y) = \hat{e}_\phi \cdot \nabla\times[X\hat{e}_\phi\times\nabla\times(Y\hat{e}_\phi)] \quad (11)$$

Again, note that the α-effect in Eq. (6) only regenerates poloidal from toroidal field. According to Eq. (7b), for $\epsilon \ll 1$ the buoyancy force drives a thermal wind

$$2\frac{\partial v_t}{\partial z} \approx \frac{\partial\Theta}{\partial\theta} \quad (12)$$

which then regenerates toroidal from poloidal field via the $M(v_t, A_0)$ ω-effect in Eq. (6b). The particular form $\Theta = -\Theta_0 r \cos^2\theta$ was chosen to facilitate comparison with the previous work of Hollerbach et al. (1992), who considered a differential rotation of the form $\omega_t = \omega_0 r$, independent of the colatitude θ. (Originally this choice of ω_t dates back to the pioneering kinematic work of Roberts (1972).) So, if we want $\omega_t = \omega_0 r$, then $v_t = \omega_0 r^2 \sin\theta$, $\partial v_t/\partial z = \omega_0 r \sin\theta\cos\theta$, and according to Eq. (12) $\Theta = -\Theta_0 r\cos^2\theta$ will generate essentially that.

In the linear, kinematic regime, the model presented here is thus essentially an $\alpha\omega$-dynamo, as derived by Braginsky (1976). In the non-linear, dynamical regime, however, there is one important difference: in the classical $\alpha\omega$-dynamo, in Eq. (7b) the $M(D^2A, A)$ contribution to the magnetic wind is traditionally neglected compared with the $M(B, B)$ contribution, on the assumption that the poloidal field is much weaker than the toroidal field. We found that this neglect had a destabilizing influence on the dynamics, and so chose to include both contributions to the magnetic wind. The stabilizing influence of including both terms has been previously noted by Barenghi (1993) and by P.H. Roberts (private communication), and is intended to be the subject of further research. Note, however, that it does have the unfortunate consequence that the exact rescaling ($A \to R_\omega^{-1/2} A$, $B \to R_\omega^{1/2} B$, see for example Eq. (2.4) of Hollerbach et al.) associated with the classical $\alpha\omega$–dynamo is no longer applicable. Thus, when we find later on that the external dipole moment of our solution is too large by a factor of 10, we unfortunately cannot simply rescale our solution appropriately to be in agreement with the Earth's observed dipole moment. The absence of this rescaling is also the reason we had to take α to be a tensor; the term we wanted to eliminate is traditionally also eliminated by this rescaling.

This completes our discussion of our geodynamo model. Further details, such as the numerical implementation, may be found in paper I, as well as in Hollerbach (1994). We turn next to the details of the new type of solution that emerged in paper II.

3. Results

As in paper II, we fix $\Theta_0 = 200$, and incrementally increase α_0. The onset of dynamo action occurs at $\alpha_0 \approx 8$, in the form of dynamo waves propagating from the equator to the pole. A second, symmetry-breaking bifurcation occurs at $\alpha_0 \approx 12$. The dynamo waves are then no longer oscillations about a zero time-average, but about a non-zero time-average. This bifurcation sequence corresponds exactly to the viscously limited solutions obtained by Hollerbach et al. (1992) in the absence of an inner core, and indeed it was the desire to compare with this initial bifurcation sequence that motivated the particular choices of α and Θ in the previous section.

So far then the presence of the inner core does not appear to be particularly significant. The primary purpose in duplicating the Hollerbach et al. initial bifurcation sequence is to emphasize that the subsequent differences really are due to a proper treatment of the inner core. After all, starting from the same initial bifurcation sequence, the Hollerbach et al. solutions subsequently degenerated into chaotic oscillations that reversed far too frequently to bear any resemblance to the geodynamo, whereas the solution

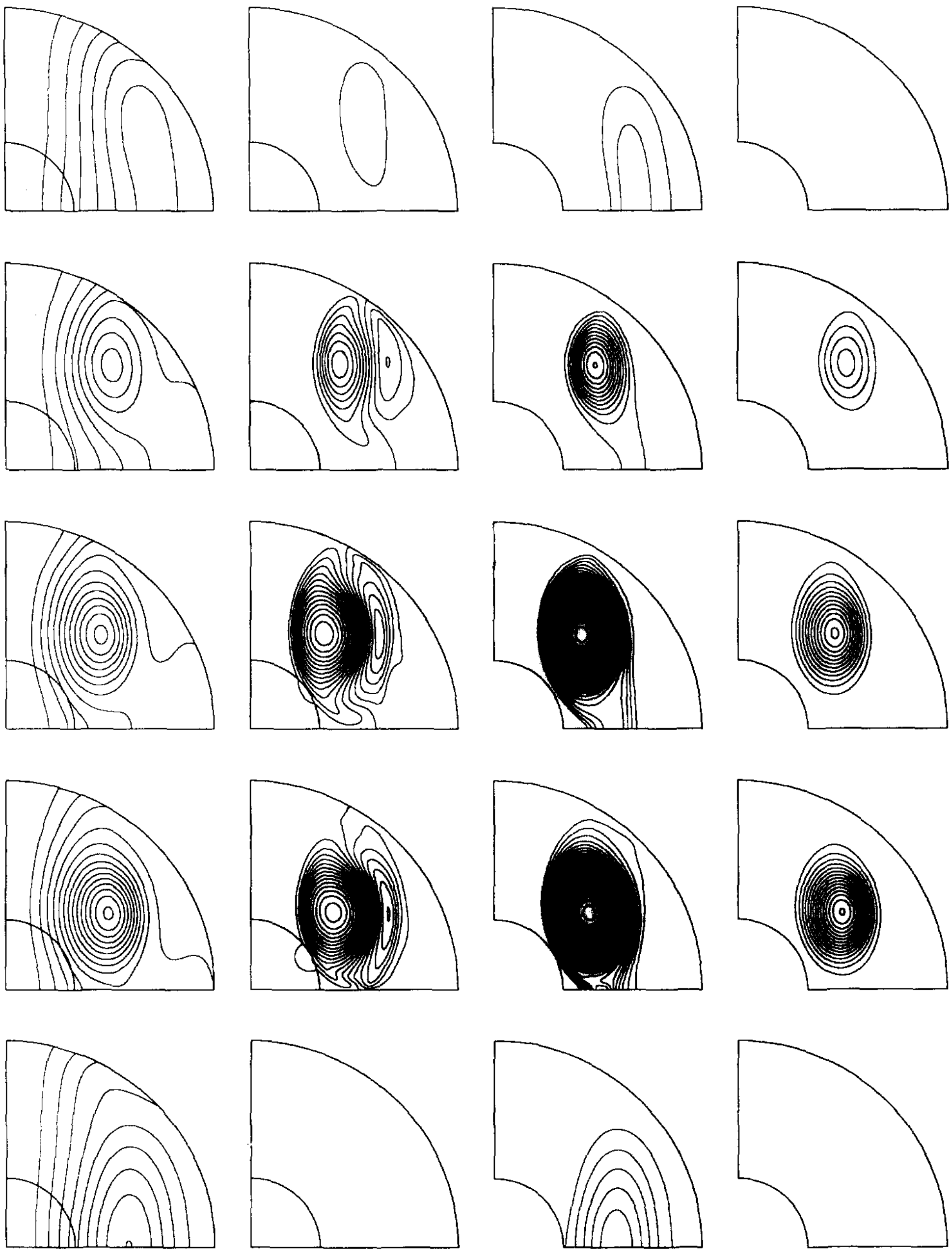

Fig. 1. From left to right, contour plots of the streamfunction of the poloidal field, the toroidal field, the angular velocity, and the streamfunction of the meridional circulation for five (from top to bottom) uniformly spaced time intervals throughout the period of 0.21, about 13 000 years in real time. Contour intervals of 1/2, 1, 50 and 5. The field and flow have been non-dimensionalized such that unity corresponds to about 16 G and 0.036 km year^{-1}, respectively. The maximum westward drift is thus about 60 km year^{-1}.

presented in paper II and here is considerably more promising.

In Fig. 1 we present one period of the resulting solution at $\alpha_0 = 50$, showing contour plots of the field and the flow at five uniformly spaced time intervals throughout the oscillation of period 0.21, corresponding to about 13000 years in real, dimensional time (see for example Roberts (1988) for the details of the non-dimensionalization). Note how virtually the entire dynamo process is confined to the region outside the inner core tangent cylinder. In paper I we pointed out that in the limit of vanishing viscosity the field had to adjust to have a vanishing electromagnetic torque on the inner core (see also Braginsky, 1964; Gubbins, 1981), and that it seemed to adjust by expelling the toroidal field from the inner core. An important feature of this new solution is that not only is the toroidal field expelled from the inner core, but from the entire region inside the inner core tangent cylinder as well.

The solution shown in Fig. 1 starts its periodic cycle with some poloidal field and zonal flow, but essentially no toroidal field or meridional flow. As time progresses, the zonal flow then gradually increases, and in the process draws out the poloidal field to generate toroidal field. Note that the maximum amplitude of the zonal flow is quite large, large enough to complete one circuit in longitude in about 1000 years or so, comparable with the observed westward drift (Jault et al., 1988; Bloxham et al., 1989), and indeed this flow is in the westward direction. Despite this large zonal flow, the toroidal field is not amplified to the enormous values one might expect; the reason is that although the zonal flow is large, its contours are so closely aligned with the contours of the poloidal field that it is very inefficient at drawing out that field. In this respect it is similar to the large geostrophic flow observed in model-Z dynamos (Braginsky and Roberts, 1987).

So, we observe in Fig. 1 a periodic solution to the mean-field geodynamo equations exhibiting very substantial, occasionally very rapid fluctuations. However, if one now focuses attention on the only feature of this solution that would be directly observable in the real geodynamo, namely the external potential field to which the poloidal

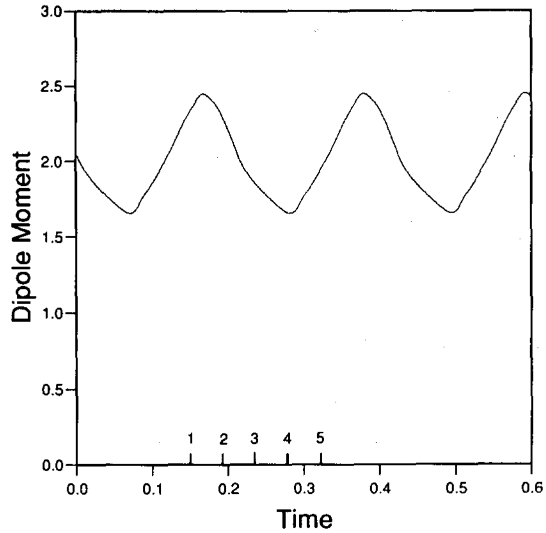

Fig. 2. The external dipole moment as a function of time. Again, the period of 0.21 is about 13000 years in real time, and the Earth's dipole moment in these units would be about 0.2. The tick marks labelled 1–5 indicate the times of the five rows in Fig. 1.

field matches, these fluctuations are not nearly so substantial. Fig. 2 shows the (non-dimensional) dipole moment of this solution as a function of time; for comparison, the Earth's dipole moment in these units would be about 0.2, so the solution obtained here is unfortunately rather large. Although the solution within the outer core undergoes fluctuations of a full order of magnitude, at the core–mantle boundary its variation is considerably less. Note also that the poloidal field lines that emerge at the core–mantle boundary are largely the same field lines that also thread the quiescent region inside the inner core tangent cylinder, that is, the structure of the field is such that the inner core and the mantle are more closely linked than one might think. This is precisely the stabilizing role we envision for the inner core; because it has a diffusive timescale of its own comparable with the timescale of the fluctuations in the outer core, it effectively averages these out to produce a much more stable external field than that typically obtained in models without an inner core. We expect that this feature is not specific to our particular model, but is likely to be common in models with a realistic treatment of the inner core. Fluctuations in the external dipole moment of the same relative magnitude, and occurring on the same 10–20 thousand year timescale as that obtained here, have been observed in the geomagnetic record (Merrill and McElhinny, 1983; Meynadier et al. 1992).

We now consider how this solution at $\epsilon = 5 \times 10^{-4}$ differs from the solution presented in paper II at $\epsilon = 10^{-3}$. Comparing Figs. 1 and 2 of paper II with Figs. 1 and 2 here, one is struck by how slight the differences are. The form of the solution is clearly at least qualitatively independent of ϵ, and so there is some hope that it may be of some geophysical interest, even though 5×10^{-4} is still an unrealistically large Ekman number. The magnitude of the poloidal field also changes very little between the two values of ϵ, but the magnitudes of the toroidal field and particularly the differential rotation increase slightly. Although these values of ϵ are still too large for us to say unambiguously that our solution is in a Taylor state, this slight increase clearly indicates that it is not in a viscously limited state (Jones, 1991).

In an attempt to understand the rapid fluctuations in the dynamically active region outside the inner core tangent cylinder, we turn to the energy balance associated with this solution. We con-

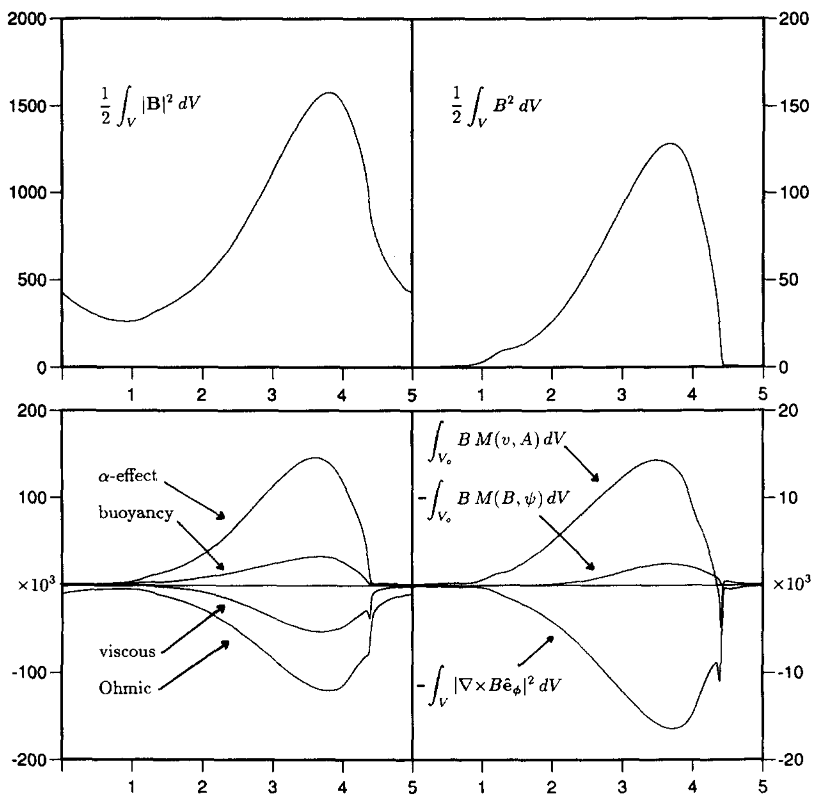

Fig. 3. Various quantities related to the two energy Eqs. (13) and (14): in the upper left panel the total magnetic energy $\frac{1}{2}\int_V |\boldsymbol{B}|^2 \mathrm{d}V$; in the lower left panel the two sources $\int_{V_0} \boldsymbol{U} \cdot \boldsymbol{r}\Theta \mathrm{d}V$ and $\int_{V_0} \boldsymbol{B} \cdot [\nabla \times (\alpha B\hat{\boldsymbol{e}}_\phi)]\mathrm{d}V - \int \alpha B_\phi B_\theta \mathrm{d}S\,|_{r=r_i}$, and the two sinks $\int_V |\nabla \times \boldsymbol{B}|^2 \mathrm{d}V$ and $\epsilon \int_{V_0} |\nabla \times \boldsymbol{U}|^2 \mathrm{d}V + \epsilon \frac{16}{3}\pi r_i^3 \Omega^2$. In the upper right panel the toroidal energy $\frac{1}{2}\int_V B^2 \mathrm{d}V$; in the lower right panel the three terms $\int_V |\nabla \times B\hat{\boldsymbol{e}}_\phi|^2 \mathrm{d}V$, $\int_{V_0} BM(v, A)\mathrm{d}V$, and $\int_{V_0} BM(B, \psi)\mathrm{d}V$. All quantities as functions of time over one period, with the tick marks labelled 1–5 corresponding to the five rows in Fig. 1.

sider first the total magnetic energy balance derived in paper I; extended to include the prescribed buoyancy force this becomes

$$\frac{1}{2}\frac{\partial}{\partial t}\int_V |\boldsymbol{B}|^2 \,\mathrm{d}V$$
$$= \int_{V_0} \boldsymbol{U}\cdot\boldsymbol{r}\Theta \,\mathrm{d}V$$
$$+ \int_{V_0} \boldsymbol{B}\cdot\left[\nabla\times(\alpha B\hat{\boldsymbol{e}}_\phi)\right] \mathrm{d}V$$
$$- \int \alpha B_\phi B_\theta \,\mathrm{d}S\,|_{r=r_i}$$
$$- \int_{V_0} |\nabla\times\boldsymbol{B}|^2 \,\mathrm{d}V - \int_{V_i} |\nabla\times\boldsymbol{B}|^2 \,\mathrm{d}V$$
$$- \epsilon\int_{V_0} |\nabla\times\boldsymbol{U}|^2 \,\mathrm{d}V - \epsilon\tfrac{16}{3}\pi r_i^3\Omega^2 \qquad (13)$$

where the volume V is all of space, V_i is the inner core, and V_0 is the outer core. Ω is the differential rotation of the inner core, determined as part of the solution by the appropriate torque balance on the inner core (Gubbins, 1981). The two sources are thus the kinematically prescribed buoyancy force and α-effect. The Ohmic dissipation throughout the entire core, and the viscous dissipation are negative-definite. The upper left panel of Fig. 3 shows the energy $\frac{1}{2}\int_V |\boldsymbol{B}|^2 \,\mathrm{d}V$, and the lower left panel shows the two source and two sink terms, both throughout one period. The time-derivative of the energy is equal to the sum of these four terms to high accuracy. The energy is in a gradual decline well before the rapid event half-way between rows four and five of Fig. 1, and aside from a brief increase in the viscous dissipation there is relatively little insight into the possible cause of this event.

So, we consider next not the total, but only the toroidal magnetic energy. After all, the toroidal field exhibits much greater fluctuations than does the poloidal field, and so this sudden collapse should figure much more prominently in the toroidal rather than in the total energy. The appropriate balance in this case is, from Eqs. (5b) and (6b),

$$\frac{1}{2}\frac{\partial}{\partial t}\int_V B^2 \,\mathrm{d}V$$
$$= -\int_V |\nabla\times B\hat{\boldsymbol{e}}_\phi|^2 \,\mathrm{d}V$$
$$+ \int_{V_0} BM(v, A)\,\mathrm{d}V - \int_{V_0} BM(B, \psi)\,\mathrm{d}V \qquad (14)$$

The upper right panel of Fig. 3 shows the toroidal energy $\frac{1}{2}\int_V B^2 \,\mathrm{d}V$, and the lower right panel shows the Ohmic sink and the two (potentially) source terms. Again, the time-derivative of the energy is indeed equal to the sum of these three terms. Also, as anticipated, this sudden collapse is much more pronounced in the purely toroidal rather than in the total energy. Furthermore, we now do have a clue as to its cause; the $\int_{V_0} BM(v, A)\mathrm{d}V$ 'source' term is in fact briefly negative, that is, the ω-effect is temporarily acting as an anti-dynamo, actively destroying the toroidal field.

In paper II we interpreted this sudden collapse as the result of a misalignment between the (generally closely aligned) zonal flow and poloidal field; the magnetic tension associated with that field would then suddenly brake the zonal flow. This interpretation is now seen to be not quite correct: after all, if the zonal flow were being brought to a halt by the magnetic tension, one would expect the $\int_{V_0} BM(v, A)\mathrm{d}V$ term to increase briefly as the toroidal field is briefly amplified. There is indeed a misalignment between various terms, but it is such that the $\int_{V_0} BM(v, A)\mathrm{d}V$ term decreases briefly, to the point where it is acting as an anti-dynamo. If anything, then, one should expect the zonal flow to be briefly amplified as it draws energy from the toroidal field, and indeed that is presumably the cause of the slight increase in the viscous dissipation noted in the lower left panel of Fig. 3.

The underlying cause of this misalignment, and the reason it generates such a rapid response, appears to be due to Taylor's constraint, requiring the integrated Lorentz torque on concentric cylindrical shells parallel to the axis of rotation to vanish (on each such cylinder) in the limit of vanishing viscosity. As noted, for example, in Hollerbach (1990), even a relatively slight misalignment from such a Taylor state will generate a large geostrophic (zonal) flow. Furthermore, the effect of this geostrophic flow on the toroidal energy is necessarily negative-definite. In fact, the drain on the toroidal energy is so unsustainably large that the system will necessarily evolve back toward a Taylor state— or otherwise end up in a viscously limited state— on a timescale that can be as rapid as $O(\epsilon^{1/2})$, in excellent agreement with the timescale observed here.

In Hollerbach et al. we noted the possibility that the system might evolve in such a manner that for almost all of the time Taylor's constraint is satisfied, so the evolution is slow and independent of ϵ, but that for a small part of the time

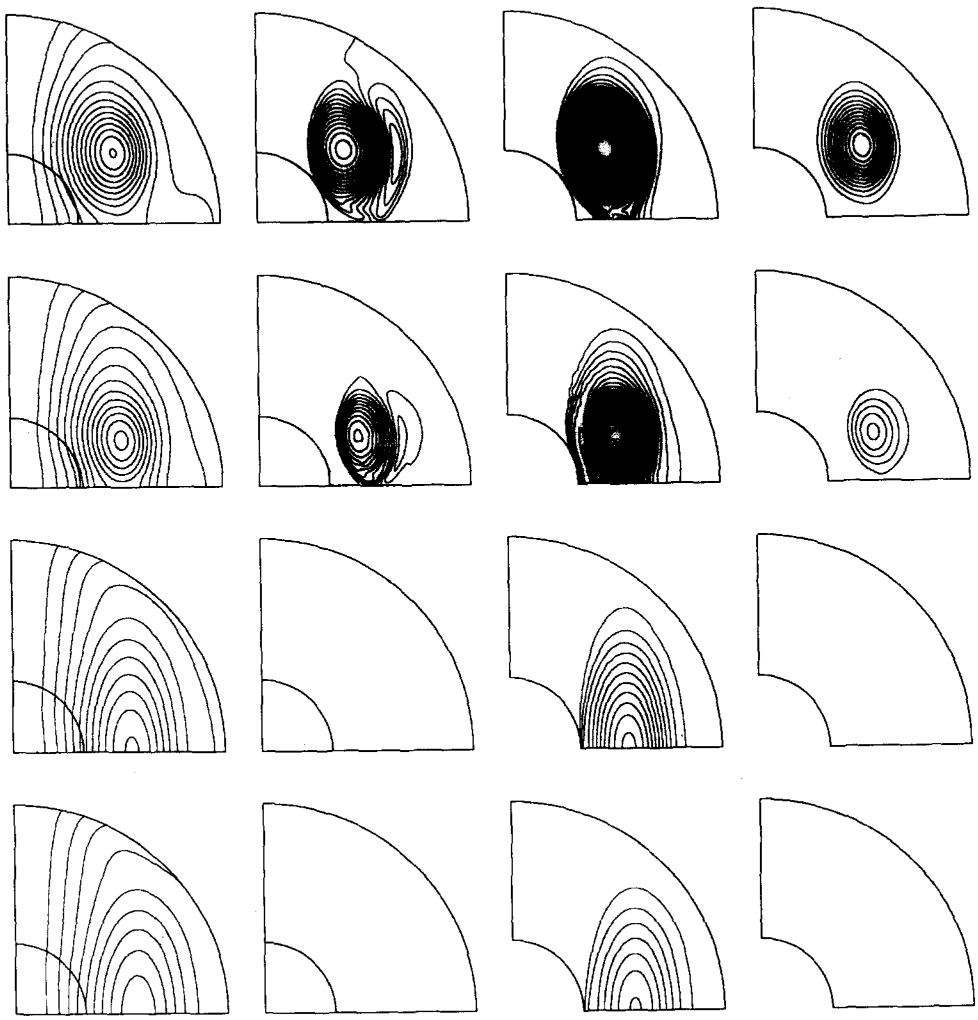

Fig. 4. As in Fig. 1, contour plots of the streamfunction of the poloidal field, the toroidal field, the angular velocity, and the streamfunction of the meridional circulation for four time intervals uniformly spaced between the fourth and fifth rows of Fig. 1. Contour intervals as in Fig. 1.

Taylor's constraint is not satisfied, so the evolution is fast and dependent on ϵ. The solution presented here would appear to be an example of such behaviour, although it must be noted that because $\epsilon = 5 \times 10^{-4}$ is still not quite in the asymptotically small regime, the distinction between the two phases is not quite so sharp.

The question then arises why certain features, such as the external dipole moment in Fig. 2, do not participate in the rapid phase of the evolution. Again, the answer is that they cannot, because of the magnetically stabilizing role of the inner core. Because a finitely conducting inner core has a diffusive timescale of its own long compared with the most rapid phase of the outer core evolution, it cannot possibly participate in this phase, but must effectively average over it. But then the strong electromagnetic coupling between inner and outer cores ensures that there must be at least some component of the outer core field that also averages over this very rapid phase. In contrast, in the spherical dynamo of Hollerbach et al., there was nothing to prevent the entire system from participating in this very rapid evolution, with disappointingly chaotic consequences. It is perhaps also worth noting that if the field oscillates about a zero time-average outside the tangent cylinder, as it does before the symmetry-breaking bifurcation, the dipole field in the inner core will be unable to build up, and solutions of this type will almost certainly not be possible.

This relative constancy of the poloidal field inside the tangent cylinder is probably the reason why the toroidal field is so weak there. In contrast to the α^2-models of paper I, where steady toroidal field is created by an α-effect, the only mechanism by which toroidal field can be created here is by the growth of dynamo waves, in which the poloidal field grows in tandem with the toroidal field. In Fig. 1 we can see that the closed poloidal field lines, which are mainly outside the tangent cylinder, and which do not leave the outer core at all, do indeed participate in the growth and decay phases of the cycle. Inside the tangent cylinder, where the long diffusion time of the inner core keeps the poloidal field fairly constant, there is no mechanism for generating toroidal field. (Note that torsional Alfven waves are excluded because there is no inertial term in the equation of motion. Since such Alfven waves have periods of only tens of years while the dynamo waves have periods of thousands of years, even if torsional oscillations were permitted they would be unlikely to be strongly excited.)

Fig. 4 shows a little more of the structure of the field during this sudden collapse; it shows the field and the flow at four time intervals uniformly spaced between the fourth and fifth rows of Fig. 1. The time interval between these rows in Fig. 4 is thus 0.01, a mere 600 years in real time. Indeed, this phase of the evolution is sufficiently rapid that one begins to wonder whether the neglect of inertia in this work is justified, particularly if this rapid timescale does scale with ϵ as suggested above.

It is evident in Fig. 4 that this collapse is associated with a sudden merger of the two maxima above and below the equator to yield a single maximum on the equator for A and v, and the annihilation of B and ψ. Remember that the imposed dipole symmetry is such that A and v are symmetric about the equator, and B and ψ are antisymmetric. In consequence, as the location of the peak value of B moves towards the equator, rapid annihilation of toroidal field occurs as oppositely directed flux moves together.

Finally, developing the ideas of Cox (1969) and Gubbins (1987), we suggested in paper II that these large fluctuations in the outer core might lead to a geomagnetic reversal, by the following mechanism: instead of an exactly periodic solution, as obtained here, imagine a quasi-periodic solution. If a larger than average fluctuation occurred, large enough and lasting long enough to reverse the field in the inner core, the whole field might reverse, but still be relatively stable both before and after. Again, in these solutions a dipole symmetry has been imposed at the outset. Hollerbach (1991) considered evolution with no particular parity imposed and found a considerably richer variety of time-dependent behaviour, so this may be just the effect needed to produce a quasi-periodic solution, particularly considering the strong coupling between the two hemispheres noted above.

4. Conclusion

That the presence of an inner core has profound consequences for core dynamics in the limit of rapid rotation is well known. For example, Busse and Cuong (1977) have demonstrated that the pattern of (non-magnetic) convection is quite different inside and outside the inner core tangent cylinder. In this work we have demonstrated that the pattern of dynamo action may be quite different inside and outside, as has also been suggested on observational evidence (Gubbins and Bloxham, 1987).

Indeed, the two issues are not distinct: since the parameters α and Θ (which we deliberately took to be smoothly distributed throughout the entire outer core so as not to stand accused of building our results in from the outset), are in fact strongly influenced by the pattern of convection, and since the pattern of convection is strongly influenced by the structure of the field, one can well imagine a mutually reinforcing effect. Only a self-consistent, convective dynamo model, which is still the ultimate objective of this project, will be fully able to address such a possibility.

What we have demonstrated in this work is that it may be very beneficial to separate the outer core into dynamically distinct regions. By strongly coupling one region to the finitely conducting inner core, and thereby preventing it from participating in the most rapid timescales of the other region, we may be able to reconcile the complicated, time-dependent nature of the field and flow in the dynamically active region with the simple, stable nature of the externally observed dipole moment of the Earth's magnetic field.

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