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On the Divergence of the Hall Current in the Westward Traveling Surge

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Recent observations suggest that the Hall current tends to be highly divergence free in the westward traveling surge. It is shown in a model calculation that the blockage of the Hall current from closure along field lines leads to a significant reduction of the divergence of the Hall current, but is not divergence free. The westward electrojet in our model is found to rotate counterclockwise around the leading edge of the surge and merge into the eastward electrojet. The difference in strength between the two electrojets at the head of the surge goes into the upward field-aligned current. It is also shown that the conductivity enhanced by the discrete auroral precipitation does not contribute to the divergence of the Hall current provided that the electric fields are coplanar along a field line. The above results strongly suggest that the Hall current tends to be highly divergence free in regions of bright auroral forms, especially in the westward traveling surge.

1. Introduction

The three-dimensional current system associated with the westward traveling surge (WTS), and more specifically the divergence of the westward electrojet at the leading edge of the surge, are fundamental issues of the electrodynamics of magnetosphere-ionosphere coupling. *Opgenoorth et al.* [1983] showed that the electric field measured by the Scandinavian Twin Auroral Radar Experiment (STARE) radar tends to be highly perpendicular to the bright auroral forms in the WTS [also, *Inhester et al.*, 1981; *Baumjohann et al.*, 1981]. Since the contours of constant brightness along auroral forms can be taken as parallel to the contours of constant conductivity, *Opgenoorth et al.*'s [1983] results can be interpreted as implying that the electric field tends to be parallel or antiparallel to the conductivity gradient (i.e., $\mathbf{E} \times \nabla\Sigma_h = 0$) in the WTS. It can be easily shown that $\mathbf{E} \times \nabla\Sigma_h = 0$ leads to $\nabla \cdot \mathbf{I}_h = 0$. Therefore it may be inferred that the divergence of the Hall current tends to be inhibited in the westward traveling surge. This interpretation also implies a fascinating possibility that the electrojet currents tend to turn into the Pedersen current near the leading edge of the surge before diverting into field-aligned currents [*Rostoker et al.*, 1975]. Recently, *Sugiura* [1984] deduced from the DE 2 satellite observations that the Hall current is highly divergence free (i.e., $\nabla \cdot \mathbf{I}_h \simeq 0$). More specifically, *Sugiura* [1984] noted that the state of divergenceless Hall current may well be a prevailing mode of the magnetosphere-ionosphere coupling.

The purpose of this paper is to show the following:

1. The observational results of *Opgenoorth et al.* [1983] can be understood quantitatively in terms of the Hall current blockage mechanism proposed by *Kan et al.* [1984].

2. The blockage of the Hall current leads to an intense upward field-aligned current at the head of the WTS associated with the divergence of the Pedersen current. The resulting Hall current is still far from divergence free.

2. Hall Current Blockage Mechanism

Blockage of the Hall current from complete magnetospheric closure can be written as

$$\nabla \cdot [\Sigma_p \mathbf{E}_p + \Sigma_h \hat{B}_0 \times (\mathbf{E}_0 + \mathbf{E}_p)] = 0 \qquad (1)$$

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where $\mathbf{E}_0$ is the electric field externally impressed on the ionosphere and $\mathbf{E}_p$ is the polarization electric field internally produced by the blockage. Note that (1) is a generalization of equation (1) of *Kan et al.* [1984] by including the Hall current of the polarization electric field. The present equation (1) states that the divergence of the Hall current of the total electric field $(\mathbf{E}_0 + \mathbf{E}_p)$ is counterbalanced by the divergence of the Pedersen current of the polarization electric field. The polarization electric field is produced by the blockage of the Hall current from closure in the magnetosphere. The amount of blockage is self-consistently determined so that the resulting polarization electric field $\mathbf{E}_p$ produces a field-aligned current to counterbalance the divergence of the Hall current driven by the applied field $\mathbf{E}_0$.

Substituting $\mathbf{E}_p = -\nabla\phi_p$ and $\mathbf{E}_0 = -\nabla\phi_0$ into (1), one obtains

$$\nabla^2\phi_p + \frac{\nabla\Sigma_p}{\Sigma_p} \cdot \nabla\phi_p + \frac{\nabla\Sigma_h}{\Sigma_p} \cdot \hat{B}_0 \times (\nabla\phi_0 + \nabla\phi_p) = 0 \qquad (2)$$

The convection pattern (i.e., equipotential contours) distorted by the polarization electric field is given by ϕ $(= \phi_p + \phi_0) = \text{const}$, corresponding to the convection streamlines. The ionospheric current and the field-aligned current can be calculated from

$$\mathbf{I}_i = \Sigma_p(\mathbf{E}_p + \mathbf{E}_0) + \Sigma_h \hat{B}_0 \times (\mathbf{E}_p + \mathbf{E}_0) \qquad (3)$$

$$J_{\parallel} = \nabla \cdot \mathbf{I}_i = \nabla \cdot (\Sigma_p \mathbf{E}_0) \qquad (4)$$

where $J_{\parallel} < 0$ for upward current flowing away from the ionosphere.

In Figure 1*a* the externally applied electric field $\mathbf{E}_0$ is assumed to be one dimensional, varying according to the tanh (x) function; in Figure 1*b* the Pedersen conductivity is assumed; and Figures 1*c* and 1*d* show the field-aligned currents due to the divergence of the Pedersen (Figure 1*c*) and the Hall (Figure 1*d*) currents driven by $\mathbf{E}_0$. The maximum value of the Pedersen conductivity is assumed to be 20 mhos, which is intended to approximate the aurora-enhanced conductivity in the westward traveling surge. The background conductivity is assumed uniform at 2 mhos. The aurora-enhanced Hall conductivity is assumed to be 3 times its Pedersen counterpart. Note that the contours of constant upward field-aligned current are shown by the dashed curves enclosing a minus sign while the downward field-aligned current is shown by the solid curves enclosing a plus sign.

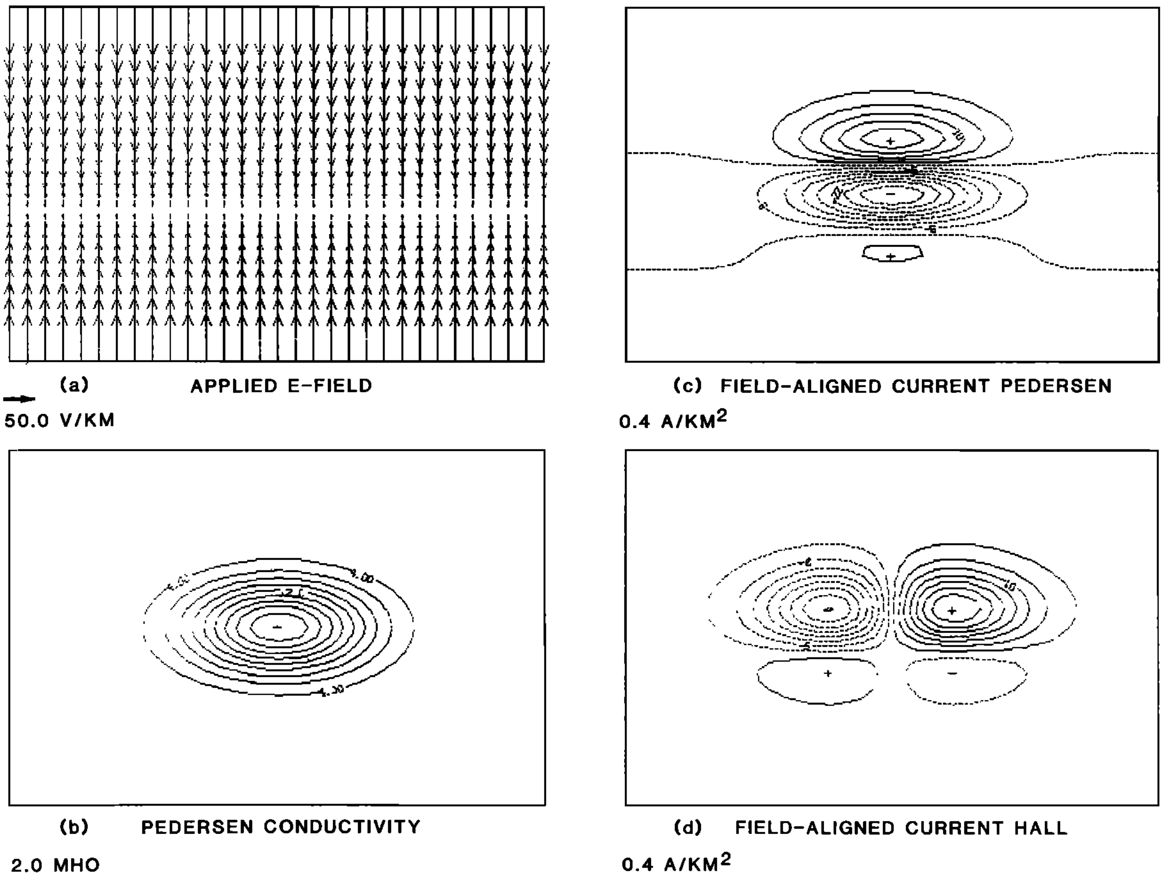

Fig. 1. (*a*) The externally imposed electric field $\mathbf{E}_0$ on the ionosphere. (*b*) The conductivity model assumed. (*c*) The field-aligned current due to the divergence of the Pedersen current driven by $\mathbf{E}_0$. (*d*) The field-aligned current due to the divergence of the Hall current driven by $\mathbf{E}_0$.

Figure 2 shows a numerical solution of (2)–(4) for the input parameters specified in Figures 1*a* and 1*b*. The boundary condition for the solution in Figure 2 is $E_n = 0$ where E_n is the normal component of the polarization electric field $\mathbf{E}_p$ on the boundary. The results are very sensitive to the boundary condition. Our choice of the boundary condition requires that the conductivity is uniform near the boundary so that the polarization electric field vanishes near the boundary. Figure 2*a* shows the total electric field $(\mathbf{E}_0 + \mathbf{E}_p)$, including the polarization electric field produced by the blockage of the Hall current described in (2). The electric field near the leading edge of the surge in Figure 2*a* clearly becomes more perpendicular to the constant conductivity contours as compared with Figure 1*a*. This perpendicularity trend is consistent with the observations by *Opgenoorth et al.* [1983]. Figure 2*b* shows the ionospheric current in the westward traveling surge. The westward electrojet can be seen to rotate counterclockwise around the leading edge of the surge and merge into the eastward electrojet. Note that a fraction of the westward electrojet diverts upward along field lines during the turning of the electrojet around the head of the surge. The results in Figure 2*b* clearly show that the eastward electrojet is closely related to the westward electrojet at the leading edge of the surge. This is consistent with the observational results of *Rostoker et al.* [1975] and others. Figure 2*c* shows the field-aligned current due to the divergence of the Pedersen current. It is clear that the Pedersen field-aligned current is upward at the leading edge of the surge. Figure 2*d*, due to the Hall current, is the field-aligned current contributed by the Hall current. From Figures 2*c* and 2*d* it is seen that $J_{\parallel \max}$ due to the divergence of the Hall current is about a half of the $J_{\parallel \max}$ due to the divergence of the Pedersen current. On the other hand, comparing Figure 2 with Figure 1*d* shows that the blockage of the Hall current does reduce the divergence of the Hall current significantly. The maximum value of $J_\parallel$ in Figure 2*d* due to the Hall current is about one-half the maximum value in Figure 1*d*. From this result we conclude that the blockage of the Hall current can reduce the divergence of the Hall current but cannot make it completely divergence free, because the blockage mechanism results in balancing the divergence of the Hall current by the divergence of the Pedersen current of the polarization field, as stated in (2).

In view of the consistency between the observational results [*Opgenoorth et al.*, 1983; *Inhester et al.*, 1981; *Baumjohann et al.*, 1981] and the theoretical results shown in Figure 2, we believe that the blockage of the Hall current from closure in the magnetosphere via field-aligned current is a fundamental process governing the electrodynamics of the magnetosphere-ionosphere coupling at the leading edge of the westward traveling surge. However, it should be kept in mind that the present model is a local model intended to describe the leading edge of the westward traveling surge. The westward and eastward electrojets inferred from such a model must be viewed as theoretical extrapolations. The correctness of a local model depends on the appropriateness of the boundary condition chosen for the solution. The results of the present local model will be tested by the global model under development.

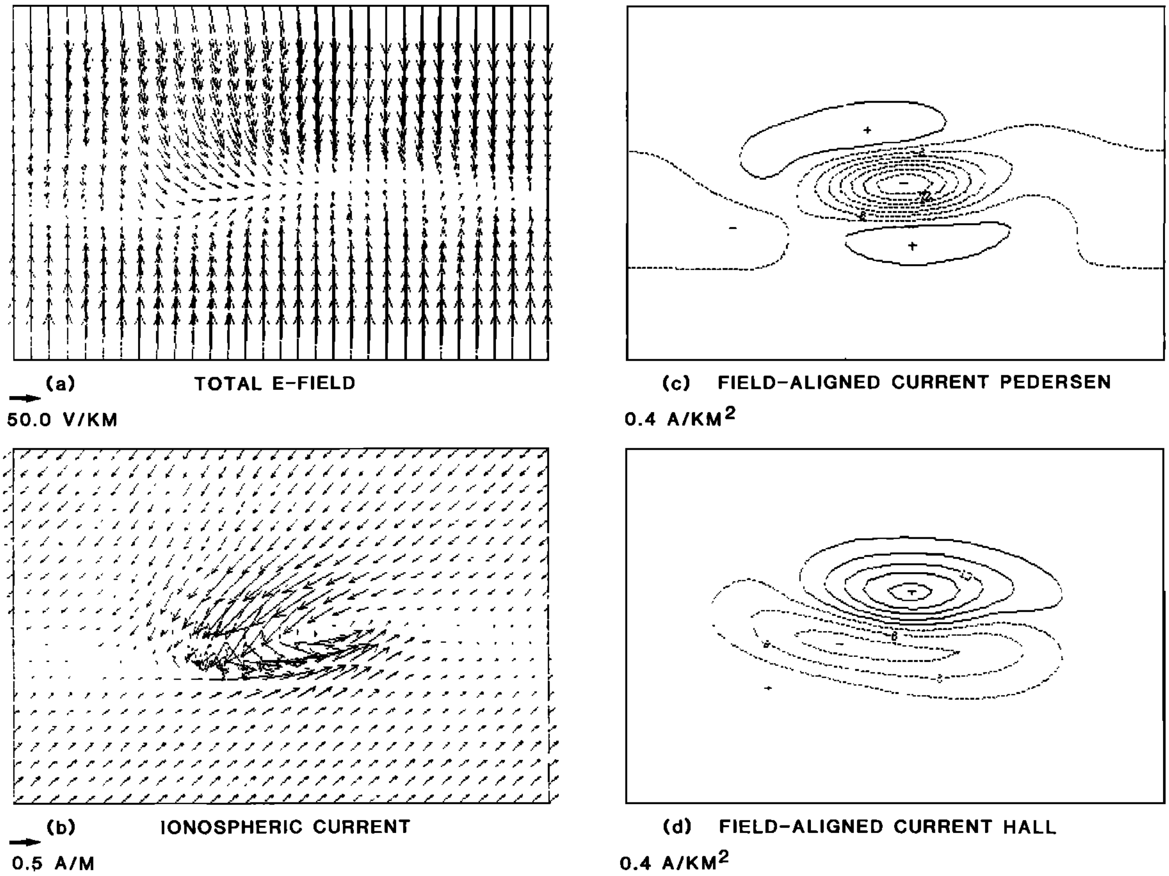

Fig. 2. (*a*) The total electric field ($\mathbf{E}_0 + \mathbf{E}_p$) including the polarization field produced by the blockage of the Hall current. (*b*) The electrojets driven by $\mathbf{E}_0 + \mathbf{E}_p$. The field-aligned current due to the divergence of (*c*) the Pedersen and (*d*) the Hall currents driven by $\mathbf{E}_0 + \mathbf{E}_p$.

3. Aurora-Enhanced Conductivity

In this section we show that the aurora-enhanced conductivity does not contribute to the divergence of the Hall current if the electric fields are coplanar along a field line.

The height-integrated ionospheric conductivites can be written approximately as [*Kan*, 1984]

$$\Sigma_{p,h} = \int_0^\infty \sigma_{p,h}\, dz = H(\sigma_{p,h})_{z_m} \quad (5)$$

$$(\sigma_p)_{z_m} = (e/B_0)(n\Omega_i/\nu_i)_{z_m} \quad (6)$$

$$(\sigma_h)_{z_m} = (e/B_0)(n)_{z_m} \quad (7)$$

where H is the effective height (altitude range) of the conducting ionosphere, the subscript z_m denotes the altitude at which the ionization production rate is maximum, ν_i is the ion-neutral collision frequency, and Ω_i is the ion gyrofrequency.

Under extreme simplifications the electron number density in a quasi-steady state can be estimated from the continuity equation, i.e.,

$$q = l \quad (8)$$

where q is the ionization product rate and l is the ionization loss rate. The ionization loss rate in the E region is dominated by the recombination process ($l \simeq \alpha_0 n^2$) rather than the attachment process ($l \simeq \alpha_1 n$) where α_0 and α_1 are proportionality constants. Thus (8) can be rewritten approximately as

$$n = (q/\alpha_0)^{1/2} \quad (9)$$

For the purpose of deriving a scaling relationship between the height-integrated conductivity and the precipitating auroral electrons, the ionization production rate q [*Rees*, 1963] can be approximated drastically by

$$q(z_m) = q_{\text{ph}} + \alpha_2 F W_0 \quad (10)$$

where q_{ph} is due to photoionization, F is the particle flux, W_0 is the energy of the precipitating particles, and α_2 is a constant depending on the model atmosphere.

Substituting (6), (7), (9), and (10) into (5), one obtains the aurora-enhanced height-integrated conductivity [*Kan*, 1984],

$$\Sigma_p = (\nu_i/\Omega_i)_{z_m}^{-1}(\Sigma_0^2 + \gamma F W_0)^{1/2} \quad (11)$$

$$\Sigma_h = (\Sigma_0^2 + \gamma F W_0)^{1/2} \quad (12)$$

where Σ_0 is the Hall or Pedersen conductivity due to photoionization and γ is a constant related to the ionization efficiency.

Note that FW_0 in (11) and (12) is the energy flux of precipitating particles summed over all species, i.e.,

$$FW_0 = \Sigma J_s[\varepsilon_{0s}/e + A(\phi_i - \phi_m)] \quad (13)$$

where $A = 1$ for the precipitating electrons in the upward field-aligned current region, $A = 0$ for the precipitating ions and electrons in the downward field-aligned current region, ε_{0s} is the thermal energy of the sth species in the source region of the precipitating particles, and J_s is the magnitude of the field-aligned current density at the ionospheric level carried by precipitating particles which can be written as [*Knight*, 1973; *Fridman and Lemaire*, 1980]

$$J_{\parallel s} = J_{0s}\left\{\frac{B_0}{B_m} - \left(\frac{B_0}{B_m} - 1\right) \cdot \exp\left[-A\,\frac{(\phi_i - \phi_m)}{(\varepsilon_{0s}/e)}\Big/\left(\frac{B_0}{B_m} - 1\right)\right]\right\} \quad (14)$$

where the parameter A has been defined in (13), $J_{0s} = n_s e(\varepsilon_{0s}/m_s)^{1/2}$ is the magnitude of the field-aligned current carried by the thermal flux inside the loss cone, and B_m is the field magnitude at the source region of field-aligned current.

Divergence of the Hall current can be written as

$$\nabla \cdot \mathbf{I}_h = \nabla\Sigma_h \cdot (\hat{B}_0 \times \mathbf{E}_i) \quad (15)$$

where $\hat{B}_0 = \mathbf{B}_0/B_0$ and $\nabla \times \mathbf{E}_i = 0$ has been assumed. The conductivity gradient can be written from (11), (12), (13), and (14) as

$$\nabla\Sigma_{h,p} = \frac{\partial\Sigma_{h,p}}{\partial(\phi_i - \phi_m)}(\mathbf{E}_m - \mathbf{E}_i) + \sum_s \frac{\partial\Sigma_{h,p}}{\partial\varepsilon_{0s}}\nabla\varepsilon_{0s} + \frac{\partial\Sigma_{h,p}}{\partial\Sigma_0}\nabla\Sigma_0 \quad (16)$$

The first term on the right-hand side of (16) is the nonuniform aurora-enhanced conductivity gradient. The second term is due to precipitating electrons and ions which contribute to the diffuse auroral luminosity. The third term is due to photoionization. In the discrete auroral region the first term dominates. In the diffuse aurora region the second term is important. The third term is important in the sunlit ionosphere but can be dropped in the nightside ionosphere.

In the discrete auroral region, especially in the leading edge of the westward traveling surge, the second and third terms in (16) can be neglected. Hence

$$\nabla \cdot \mathbf{I}_h = \frac{\partial\Sigma_h}{\partial(\phi_i - \phi_m)}(\mathbf{E}_m - \mathbf{E}_i) \cdot \hat{B}_0 \times \mathbf{E}_i \quad (17)$$

It follows from (17) that the aurora-enhanced Hall current is divergence free if the electric fields $\mathbf{E}_m$ and $\mathbf{E}_i$ are coplanar because $\mathbf{E}_i \cdot \hat{B}_0 \times \mathbf{E}_i$ is identically zero. This conclusion is independent of the exact functional form of the aurora-enhanced Hall conductivity; i.e., Σ_h can be any function of ε_{0s}, Σ_0, and $(\phi_i - \phi_m)$.

The coplanarity requirement on $\mathbf{E}_m$ and $\mathbf{E}_i$ is satisfied in a steady state when $\nabla \times \mathbf{E} = 0$. The existing electrostatic models of the auroral acceleration region all satisfy the coplanarity condition. However, one cannot rule out the possibility that $\mathbf{E}_m$ and $\mathbf{E}_i$ may become noncoplanar along an auroral field line in a nonsteady state situation.

4. Summary and Conclusion

In section 2 we showed that the electric fields measured by the STARE radar in the westward traveling surge [*Opgenoorth et al.*, 1983] are consistent with the blockage of the Hall current from closure in the magnetosphere along field lines. As a consequence of the blockage, the westward electrojet is found to rotate counterclockwise around the leading edge of the surge and merge into the eastward electrojet. At the same time, the divergence of the Hall current is much reduced at the leading edge of the surge. However, our results show that the Hall current is far from completely divergence free. In section 3 we showed that the aurora-enhanced conductivity can be expected to make no contribution to the divergence of the Hall current in the westward traveling surge. The results in sections 2 and 3 strongly suggest that the Hall current tends to be highly but not completely divergence free, especially in regions of bright auroral forms as exemplified in the westward traveling surge.

In conclusion, we believe that the blockage of the Hall current from closure in the magnetosphere along field lines is a governing process in the electrodynamics of the magnetosphere-ionosphere coupling, especially in the WTS. As a consequence of the blockage process, the eastward electrojet is found to be closely related to the westward electrojet at the leading edge of the westward traveling surge. Further theoretical analyses are needed to determine whether or not the Hall current can be highly divergence free at all local times around the auroral oval.

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