1035 Example Sheet 1.

To be handed in on Friday 12th October. Note that when a question involves fractions I expect exact answers as fractions and will not accept decimal approximations (e.g. 0.33 is not an acceptable substitute for 1/3).

1. **Perform complex additions.**
   (a) $(11+23i)+(5+6i)$
   (b) $(-3+4i)+(6+7i)$
   (c) $(7-2i)+(-1-i)$
   (d) $(9+i)+(11-i)$
   (e) $(6+3i)+(2.3+2.3i)$
   (f) $(\frac{1}{10}+\frac{2}{5}i)+(3.2-2.6i)$
   (g) $(210-45i)+(23-22i)$
   (h) $(-1-i)+(1-3i)$
   (i) $(\frac{1}{6}-\frac{1}{7}i)+(\frac{1}{7}+\frac{1}{6}i)$
   (j) $(2.73-3.23i)+(1.24+4.26i)$.

2. **Perform complex subtractions.**
   (a) $(11+23i)-(5+6i)$
   (b) $(-11+23i)-(5-6i)$
   (c) $(-3-11i)-(7-6i)$
   (d) $(-2-i)-(-2-i)$
   (e) $(34+22i)-(32-12i)$
   (f) $(-11.3-23i)-(2.7-23.2i)$
   (g) $(31.12-22.12i)-(43.21+55.23i)$
   (h) $(\frac{1}{5}-\frac{1}{3}i)-(\frac{1}{4}+\frac{1}{7}i)$
   (i) $(-\frac{1}{2}-\frac{1}{2}i)-(\frac{2}{3}-\frac{5}{3}i)$

3. **Perform complex multiplications.**
   (a) $i\times(-3i)$
   (b) $7\times(23-16i)$
   (c) $(3-2i)(2+2i)$
   (d) $(4+5i)(4-5i)$
   (e) $(4+5i)(5i-4)$
   (f) $(-1-i)(-8+i)$
   (g) $(3.2+1.5i)(6+i)$
   (h) $(1+i)(1+2i)(1+3i)$
   (i) $(\frac{1}{2}+\frac{1}{4}i)^2$
   (j) $\frac{2+i}{3}\times\frac{1+2i}{5}$

4. **Perform complex divisions.**
   (a) $\frac{1}{3+2i}$
   (b) $\frac{2+i}{1-i}$
   (c) $\frac{(1+i)(2-i)}{(3-i)(2+i)}$
   (d) $\frac{2i+4}{3-i}$
   (e) $\frac{1/(1+i)}{1/(2+i)}$
   (f) $\frac{1.2+3.6i}{0.2-0.2i}$
   (g) $\frac{\frac{1}{2}-\frac{1}{4}i}{\frac{1}{2}-\frac{1}{3}i}$
   (h) $\frac{11-4i}{2i-3}$
   (i) $\frac{(1+i)(2+i)}{i(3+i)}$

5. Find the modulus of each complex number (exact answers please, e.g. $\sqrt{3}$rather than 1.732).
   (a) -11
   (b) $1-i$
   (c) $\frac{3-4i}{5}$
   (d) $-11i$
   (e) $1+\sqrt{2}i$
   (f) $\sqrt{2}+\sqrt{3}i$
   (g) $\frac{1}{1+i}$
   (h) $7+6i$
   (i) $\frac{6+8i}{3}$
   (j) $\frac{1}{2}+\frac{1}{5}i$

6. Find the principal value of the argument of each complex number in radians.
   (a) 17
   (b) -17
   (c) $17i$
   (d) $-17i$
   (e) $1+i$
   (f) $\sqrt{3}+i$
   (g) $-\sqrt{3}+i$
   (h) $17i(\sqrt{3}+i)$

(i) $(1+i)^{44}$

(j) $\cos(\pi/5) + i\sin(\pi/5)$

7. Find integer powers of complex numbers.

(a) i^4

(b) i^{44}

(c) $(1+\sqrt{3}i)^3$

(d) $(1+\sqrt{3}i)^9$

(e) $(2+\sqrt{3}i)^{-1}$

(f) $(2+\sqrt{3}i)^{-2}$

(g) $(1-i)^2$

(h) $(1+i)^8$

(i) $(\frac{1+i}{\sqrt{2}})^{10000}$

(j) $(4-i)^{-2}$

8. Find all complex solutions to the equations

(a) $z^2 - z + 1 = 0$

(b) $z^2 + 9 = 0$

(c) $z^3 - 1 = 0$

(d) $z^2 = i$

(e) $z^4 + 9 = 0$

(f) $z^4 + 5z^2 + 6 = 0$

(g) $z^4 - z^2 - 6 = 0$

9. Draw on the Argand diagram the set of all complex numbers z such that

(a) $|z| = 3$

(b) $|z - 2 + i| = 3$

(c) $z^6 = 1$

(d) $(z - i + 1)^6 = 1$

(e) $\arg(z) = \pi/3$

(f) $\arg(z - 2i) = \pi/3$

(g) $|z-1| = |z-5|$

(h) $|z-1| + |z-5| = 6$

(i) $|z-i| + |z-5i| = 6$

(j) $\arg(\frac{z}{z-i}) = \pi/2$.

10. It is rumoured (and optimistically, it may even be proved by Prof. Dales in the latter stages of this course) that the exponential function $e^x$is equal to the (in some sense) convergent sum $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. It is likewise rumoured that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$, and that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$. Assuming these facts, and assuming it is valid to rearrange these infinite sums any way you like, show that we will also have

$$e^{ix} = \cos x + i\sin x.$$