MATH1050 : SOLUTIONS 5

1. (i) $s_n = (2 - 4/n^3)/(3 + 6/n^2 + 1/n^3) \to 2/3$ since $1/n \to 0$.

(ii) $s_n = (2/n - 1/n^2)/(1 + 6/n^2) \to 0/1 = 0$.

(iii) Converges to zero by squeeze test $(-1/n \le s_n \le 1/n)$.

(iv) Diverges to plus infinity.

(v) Diverges.

(vi) Converges to zero by squeeze test $(0 \le s_n \le 2^{-n})$.

2. (a) $\sum_{n=1}^{\infty}(1/n - 1/(n+1)) = 1$ as in lectures.

(b) $\sum_{n=1}^{\infty}(1/n - 1/(n+3))/3 = (1 + 1/2 + 1/3)/3 = 11/18$.

(c) G.P., sum is $1/(e \cdot (1 - 1/e)) = 1/(e-1)$ as in lectures.

(d) G.P., ratio has modulus $1/\sqrt{2} < 1$ so sum is convergent, to $1/(2/(1+i) - 1) = (1+i)/(1-i) = i$.

(e) Real part of a complex GP, or sum of two complex GPs, converges to real part of $e^{i\theta}/2 \cdot 1/(1 - e^{i\theta}/2)$, which is $2\cos\theta/(5 - 4\cos\theta)$.

(f) Imaginary part of a complex GP, or difference of two complex GPs, converges to imaginary part of $e^{i\theta}/4 \cdot 1/(1 - e^{i\theta}/4)$, which is $4\sin\theta/(17 - 8\cos\theta)$.

3. (a) Converges by comparison with $\sum 1/n^2$.

(b) Converges by ratio test.

(c) Diverges since a_n doesn't tend to zero.

(d) Converges by ratio test or by comparison with $\sum 2^{-n}$ etc.

(e) Converges by integral test.

(f) Diverges by comparison with $1/(n+1)\log(n+1)$ and integral test.

(g) Diverges by comparison with $1/n\log(n)^2$ and integral test.

(h) Diverges since a_n doesn't tend to zero.

(i) Converges by ratio test.

4. (i) From the previous homework, $d/dx(\arctan(\sinh x)) = \text{sech} x$, so $\int \text{sech} x dx = \arctan(\sinh x) + c$.

(ii) $\sec y$.

(iii) The substitution gives

$$\int_{x=0}^{\infty} \frac{\log x \ dx}{1+x^2} = -\int_{y=0}^{\infty} \frac{\log y \ dy}{1+y^2}$$

hence the integral is zero.

(iv) $\frac{\partial f}{\partial t} = -k^2 e^{-k^2 t} \cos(kx) = \frac{\partial^2 f}{\partial x^2}$ so the heat equation is indeed satisfied.