

MATH1050 : EXAMPLES 5

Hand in on Tuesday 28th November

1. Investigate convergence of the following sequences $\{s_n\}$. If the sequence converges, find its limit. If not, determine if it $\rightarrow +\infty$ or $\rightarrow -\infty$ (or neither).

(i) $s_n = \frac{2n^3 - 4}{3n^3 + 6n + 1},$

(ii) $s_n = \frac{2n - 1}{n^2 + 6},$

(iii) $s_n = \frac{\sin n\pi/6}{n},$

(iv) $s_n = 6^n,$

(v) $s_n = (-6)^n,$

(vi) $s_n = 2^{-n}$ (if n is odd), or 0 (if n is even).

2. Find the sums of the following infinite series. (You may like to try summing a few terms on your tame computer to see if you have the right answers)

(a) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. (b) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

(c) $\sum_{n=1}^{\infty} e^{-n}$. (d) $\sum_{n=1}^{\infty} (1+i)^n 2^{-n}$

(e) $\sum_{n=1}^{\infty} 2^{-n} \cos n\theta$. (f) $\sum_{n=1}^{\infty} 4^{-n} \sin n\theta$

3. Test these series for convergence, saying which test is used.

(a) $\sum_{n=1}^{\infty} \frac{n^3}{n^5+n+6}$, (b) $\sum_{n=1}^{\infty} ne^{-n}$, (c) $\sum_{n=1}^{\infty} n$,

(d) $\sum_{n=1}^{\infty} \frac{1}{2^n(n+1)(n+2)}$, (e) $\sum_{n=1}^{\infty} \frac{1}{n^{1.001}}$, (f) $\sum_{n=1}^{\infty} \frac{1}{n \log(n+1)}$,

(g) $\sum_{n=1}^{\infty} \frac{1}{n(\log(n+1))^2}$, (h) $\sum_{n=1}^{\infty} 1 + \sin(n\pi/4)$, (i) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \dots (2n-1)}{1 \cdot 4 \cdot 7 \cdot 10 \dots (3n-2)}$.