

MATH1050 : EXAMPLES 6
Hand in on Tuesday 11th December

1. Test the following series for convergence or divergence, stating in each case which test is used.

$$\begin{aligned} \text{(i)} \quad \sum_1^{\infty} \frac{2^n n!}{n^n}, & \quad \text{(ii)} \quad \sum_2^{\infty} \frac{1}{\sqrt{n^2 - 1}}, \\ \text{(iii)} \quad \sum_1^{\infty} \frac{(-1)^{n+1}}{n^4}, & \quad \text{(iv)} \quad \sum_1^{\infty} \frac{(-1)^n}{\sqrt{n}}. \end{aligned}$$

2. For the following power series find the radius of convergence, and discuss convergence at the end points of the (real) interval:

$$\text{(i)} \quad \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n \quad \text{(ii)} \quad \sum_{n=1}^{\infty} \frac{(2n)!}{n!} x^n \quad \text{(iii)} \quad \sum_{n=0}^{\infty} \frac{n^n}{n! + 1} \quad \text{(iv)} \quad \sum_{n=0}^{\infty} \frac{2^n x^n}{n + 1}.$$

3. Find the terms of the Maclaurin series for $\frac{1}{(x+1)^2}$, as far as the term in x^3 . Give an expression for the remainder R_4 .

4. Find the terms of the Taylor series with centre 9 for \sqrt{x} as far as the term in $(x-9)^2$, giving an expression for the remainder R_3 .

Use these terms to obtain an approximation for the value of $\sqrt{10}$, and to estimate the error in your approximation.