Exercise 5

This exercise sheet is not to be handed in, and will not be marked. However, the material on it is just as important as the other exercise sheets.

Integration (Week 9 work)

1. Find the following indefinite integrals:

a) $\int 5x^4 + 8x \; dx$;

b) $\int \sqrt{x} + \frac{6}{x^2} \; dx$;

c) $\int (x^2 + 3)^2 \; dx$;

d) $\int \cos x + 3 \; dx$.

Hence find the following definite integrals:

e) $\int_0^2 5x^4 + 8x \; dx$;

f) $\int_1^3 \sqrt{x} + \frac{6}{x^2} \; dx$;

g) $\int_{-1}^1 (x^2 + 3)^2 \; dx$;

h) $\int_0^{\pi/2} \cos x + 3 \; dx$.

2. Using the indicated substitutions, find the following integrals.

a) $\int \sin(2x) \; dx \; (u = 2x)$;

b) $\int e^{x/2} \; dx \; (u = x/2)$;

c) $\int \frac{1}{2x+1} \; dx \; (u = 2x + 1)$;

d) $\int x \cos(x^2 - 1) \; dx \; (u = x^2 - 1)$;

e) $\int x(x + 3)^8 \; dx \; (u = x + 3)$;

f) $\int \cos x \sin^2 x \; dx \; (u = \sin x)$;

g) $\int_0^{\pi/4} \sec^2 x \tan^3 x \; dx \; (u = \tan x)$;

h) $\int_0^1 x\sqrt{1 - x^2} \; dx \; (u = 1 - x^2)$.

3. Find the following integrals using integration by parts.

a) $\int xe^x \; dx$;

b) $\int x^2 e^x \; dx$;

c) $\int x \ln x \; dx$;

d) $\int x^3 \ln x \; dx$;

e) $\int_2^3 \frac{\ln x}{x^2} \; dx$;

f) $\int_0^{\pi} x \sin x \; dx$.

4. Express $\frac{5x+3}{(x+1)(2x+1)}$ in partial fractions. Hence find

$$\int \frac{5x+3}{(x+1)(2x+1)} \, dx,$$

and show that

$$\int_0^1 \frac{5x+3}{(x+1)(2x+1)}\,dx = \ln(4\sqrt{3}).$$

5. Find the following indefinite integrals.

a) $\displaystyle\int \frac{x^5}{x^3-1}\,dx$;

b) $\displaystyle\int \cos^2 x\,dx$;

c) $\displaystyle\int \cos^3 x\,dx$;

d) $\displaystyle\int \frac{\sin^3 x}{1+\cos x}\,dx$.

Ordinary differential equations (Week 10 work)

6. Find the general solutions of the following differential equations.

a) $\dfrac{dy}{dx} = 4y$;

b) $\dfrac{dy}{dx} = (x+2)y$;

c) $\dfrac{dy}{dx} = e^{-y}\cos x$.

Hence find the particular solutions which satisfy the following conditions.

d) $\dfrac{dy}{dx} = 4y$ with $y = 3$ when $x = 0$;

e) $\dfrac{dy}{dx} = (x+2)y$ with $y = 2$ when $x = 0$;

f) $\dfrac{dy}{dx} = e^{-y}\cos x$ with $y = 1$ when $x = \pi/2$.