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FORMULA SHEET ATTACHED

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Examination for the degree of

**BEng/ MEng
Civil Engineering****PREPARATORY MATHEMATICS
FOR ENGINEERS****Time allowed: 2 hours**

Attempt 4 questions

1. (a) Find the quotient and remainder when the polynomial $x^4 - x^3 + 2x - 1$ is divided by $x^2 - 2x$. Hence write

$$\frac{x^4 - x^3 + 2x - 1}{x^2 - 2x}$$

as the sum of a polynomial and a proper rational function.

[6 marks]

- (b) Let $f(x) = x^3 + ax^2 + bx - 2$. Suppose that $x - 2$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $x + 3$ is 1. Find the values of a and b .

[6 marks]

- (c) Express $\frac{2x+7}{(x-1)(x+2)^2}$ in partial fractions.

[8 marks]

2. (a) (i) Find the equation of the circle with centre at $A = (1,2)$ which passes through the point $B = (3,5)$.
- (ii) Find the gradient of the line joining A and B .
- (iii) Find the gradient and the equation of the tangent to the circle at B .

[12 marks]

- (b) (i) Expand the expression $(3x - 2y)^3$.
- (ii) How many arrangements are there of the letters of the following words?
a) FOUR, b) TWELVE.

[8 marks]

3. (a) Write the expression $4 \cos \theta + \sin \theta$ in the form $r \sin(\theta + \alpha)$, for suitable constants r and α , giving both to 3 significant figures, and giving α in degrees. Hence, or otherwise, find all angles $0^\circ \leq \theta \leq 360^\circ$ for which

$$4 \cos \theta + \sin \theta = 3.$$

[8 marks]

- (b) Find a formula for $\sin 3\theta$ in terms of $\sin \theta$.

[6 marks]

- (c) The value $\pounds V$ of a certain car is modelled by the equation

$$V = P e^{-kt},$$

where P and k are positive constants, and t is the time elapsed in years since the car was new.

- (i) The value of the car when new was $\pounds 7,000$, and when it was exactly two years old, it was $\pounds 4,000$. Find the values of P and k , giving both to 4 significant figures.
- (ii) What was the value, to the nearest pound, of the car when it was exactly four years old?

[6 marks]

4. (a) Find $\frac{dy}{dx}$ in each of the following cases:

(i) $2x^2 - \frac{3}{\sqrt{x}}$;

(ii) $y = \cos 3x - x \sin x$;

(iii) $y = e^{2x} \ln(\tan x)$;

(iv) $y = \sin^{-1} 2x$.

[12 marks]

(b) Let C be the curve with equation

$$x^2 + y^2 + xy - 7 = 0.$$

Find the equation of the tangent to the curve C at the point $(1,2)$.

[6 marks]

(c) For the parametric curve $x = \cos t$, $y = \sin 2t$, find an expression for $\frac{dy}{dx}$ in terms of the parameter t .

[2 marks]

5. (a) Using the substitution $u = x^2 + 2$, find the integral

$$\int x \cos(x^2 + 2) dx.$$

[6 marks]

(b) Using integration by parts, evaluate the definite integral

$$\int_{1/2}^1 x \ln 2x dx.$$

[6 marks]

(c) Find the general solution to the differential equation

$$\frac{dy}{dx} = 2xe^y.$$

Find also the particular solution which satisfies the initial condition that $y = 0$ when $x = 3$.

[8 marks]

Trigonometry

$$\sin(-\theta) = -\sin \theta, \quad \cos(-\theta) = \cos \theta.$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \operatorname{cosec} \theta = \frac{1}{\sin \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}.$$

$$\sin^2 \theta + \cos^2 \theta = 1, \quad \tan^2 \theta + 1 = \sec^2 \theta, \quad \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta.$$

$$y = \sin^{-1} x \text{ means } x = \sin y \text{ and } -90^\circ \leq y \leq 90^\circ.$$

$$y = \cos^{-1} x \text{ means } x = \cos y \text{ and } 0^\circ \leq y \leq 180^\circ.$$

$$y = \tan^{-1} x \text{ means } x = \tan y \text{ and } -90^\circ < y < 90^\circ.$$

Alternative notation: $\arcsin x = \sin^{-1} x$, $\arccos x = \cos^{-1} x$, $\arctan x = \tan^{-1} x$.

Note that $\sin^{-1} x \neq (\sin x)^{-1}$, $\cos^{-1} x \neq (\cos x)^{-1}$, $\tan^{-1} x \neq (\tan x)^{-1}$.

However $\sin^2 x = (\sin x)^2$, $\cos^2 x = (\cos x)^2$, $\tan^2 x = (\tan x)^2$.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B.$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}.$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$a \cos \theta + b \sin \theta = r \sin(\theta + \alpha)$ where $r = \sqrt{a^2 + b^2}$ and α is the angle of the point on the unit circle with coordinates $(b/r, a/r)$.

Logarithms

$$\log_a x = p \text{ means } x = a^p, \quad \log_a 1 = 0, \quad \log_a a = 1.$$

$$\log_a x + \log_a y = \log_a(xy), \quad \log_a x - \log_a y = \log_a\left(\frac{x}{y}\right), \quad k \log_a x = \log_a(x^k).$$

$$\ln x = \log_e x, \quad a^x = e^{x \ln a}.$$

Differentiation

$$\text{If } y = u \pm v, \text{ then } \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}. \quad \text{If } y = au \text{ with } a \text{ constant, then } \frac{dy}{dx} = a \frac{du}{dx}.$$

$$\text{If } y = uv, \text{ then } \frac{dy}{dx} = \frac{du}{dx} v + u \frac{dv}{dx}. \quad \text{If } y = \frac{u}{v}, \text{ then } \frac{dy}{dx} = \frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}.$$

$$\text{If } y \text{ is a function of } u, \text{ where } u \text{ is a function of } x, \text{ then } \frac{dy}{dx} = \frac{du}{dx} \frac{dy}{du}.$$

Integration

If $y = u \pm v$, then $\int y dx = \int u dx \pm \int v dx$. If $y = au$ with a constant, then $\int y dx = a \int u dx$.

If u is a function of x then $\int y dx = \int y \frac{dx}{du} du$. $\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx$.

Standard derivatives and integrals

$y = f(x)$	$\frac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{x^2+1}$

$y = f(x)$	$\int y dx$
x^n	$\frac{x^{n+1}}{n+1} + c$ ($n \neq -1$)
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\tan x$	$-\ln \cos x + c$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + c$
$\frac{1}{x^2+1}$	$\tan^{-1} x + c$