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Solutions

1. a) By long division we get the quotient $x^2 + x + 2$ and the remainder $6x - 1$. Hence

$$\frac{x^4 - x^3 + 2x - 1}{x^2 - 2x} = x^2 + x + 2 + \frac{6x - 1}{x^2 - 2x} \,.$$

b) We know that $0 = f(2) = 8 + 4a + 2b - 2$ and that $1 = f(-3) = -27 + 9a - 3b - 2$. By solving these two equations we get $a = 7/5$ and $b = -29/5$.

c) We try to find A, B and C so that

$$\frac{2x + 7}{(x - 1)(x + 2)^2} = \frac{A}{x - 1} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2} \,.$$

Clearing the fractions shows that we must have

$$2x + 7 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1).$$

By putting $x = 1$ we see that $A = 1$. Putting $x = -2$ we see that $C = -1$. Then, equating coefficients of x^2 gives $A + B = 0$ so that $B = -1$. Hence,

$$\frac{2x + 7}{(x - 1)(x + 2)^2} = \frac{1}{x - 1} + \frac{-1}{x + 2} + \frac{-1}{(x + 2)^2} \,.$$

2. a) (i) The radius of the circle is $\sqrt{(3 - 1)^2 + (5 - 2)^2} = \sqrt{13}$. Hence the circle has equation

$$(x - 1)^2 + (y - 2)^2 = 13.$$

(ii) The gradient of the line joining A and B is $(5 - 2)/(3 - 1) = 3/2$.

(iii) The tangent at B is perpendicular to AB and so has gradient $-2/3$. Its equation is thus $y - 5 = (-2/3)(x - 3)$, i.e., $y = (-2/3)x + 7$.

b) (i) By the Binomial Theorem we get

$$\begin{aligned}(3x - 2y)^3 =& (3x)^3 + 3(3x)^2(-2y) + 3(3x)(-2y)^2 + (-2y)^3 \\ =& 27x^3 - 54x^2y + 36xy^2 - 8y^3.\end{aligned}$$

(ii) a) Since no repeated letters, there are $4! = 24$ arrangements of FOUR.
b) Since E is repeated, there are $6!/2 = 360$ arrangements of TWELVE.

3. **a)** $4\cos\theta + \sin\theta = \sqrt{17}(\frac{4}{\sqrt{17}}\cos\theta + \frac{1}{\sqrt{17}}\sin\theta) = r\sin(\theta+\alpha)$ where $r = \sqrt{17} \approx 4.12$ and $\alpha = \tan^{-1}(4) \approx 76.0°$. The equation now becomes

$$\sqrt{17}\sin(\theta+\alpha) = 3,$$

which has the solutions

$$\theta + \alpha = \sin^{-1}(\frac{3}{\sqrt{17}}) + n360° \approx 46.7° + n360° \text{ and } 180° - 46.7° + n360°,$$

so that $\theta = -29.3° + n360°$ or $180° - 122.7° + n360°$. In the specified interval we have the two solutions $\theta \approx -29.3° + 360° = 330.7°$ and $\theta = 180° - 122.7° = 57.3°$.

b) $\sin(3\theta) = \sin(2\theta + \theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta = 2\sin\theta\cos^2\theta + 2(\cos^2\theta - \sin^2\theta)\sin\theta = 2\sin\theta(1-\sin^2\theta) + (1-2\sin^2\theta)\sin\theta = 3\sin\theta - 4\sin^3\theta$.

c) (i) When $t = 0$ we have $V = 7000$ so that $7000 = Pe^0 = P$. When $t = 2$ we have $V(2) = 4000$, i.e. $4000 = 7000e^{-2k}$, which gives $k = -(1/2)\ln(4/7) \approx 0.2798$.

(ii) When $t = 4$ we have $V = 7000e^{-0.2798\times 4} \approx 2286$ pounds.

4. **a)** (i) $\frac{dy}{dx} = 4x + \frac{3}{2x^{3/2}}$;

(ii) $\frac{dy}{dx} = -3\sin 3x - x\cos x - \sin x$;

(iii) $\frac{dy}{dx} = 2e^{2x}\ln\tan x + e^{2x}\sec x \operatorname{cosec} x$;

(iv) $\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$.

b) By taking the derivative of the equation of the curve we get

$$0 = 2x + 2yy' + xy' + y = 0$$

so that

$$y' = -\frac{2x+y}{2y+x}.$$

Hence at $(1,2)$, $y' = -4/5 =$ gradient of tangent at $(1,2)$, so equation of tangent is $(y-2)/(x-1) = -4/5$, i.e., $y = (-4/5)x + 14/5$.

c) $x = \cos t$, $y = \sin 2t$ so $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{2\cos 2t}{-\sin t} = -2\frac{\cos 2t}{\sin t}$.

5. **a)** Put $u = x^2 + 2$ then $du = 2x\,dx$ so that

$$\int x\cos(x^2+2)dx = \int \frac{1}{2}\cos u\,du = \frac{1}{2}\sin u + c\,.$$

b) Integrating by parts,

$$\int_{1/2}^{1} x\ln 2x\,dx = \left[\frac{1}{2}x^2\ln 2x\right]_{1/2}^{1} - \int_{1/2}^{1} 2\frac{1}{2x}\frac{1}{2}x^2 dx = \frac{1}{2}\ln 2 - \frac{1}{4}\left[x^2\right]_{1/2}^{1} = \frac{1}{2}\ln 2 - \frac{3}{16}\,.$$

c) We separate the variables and get the equation

$$e^{-y}dy = 2xdx.$$

Integrating gives

$$-e^{-y} = x^2 + \text{const}, \text{ i.e., } e^{-y} = c - x^2$$

and so the general solution is

$$y = -\ln(c - x^2)$$

where c is a constant. Putting $x = 3$, $y = 0$ gives $c = 10$, so the particular solution satisfying the given initial condition is

$$y = -\ln(10 - x^2)\,.$$