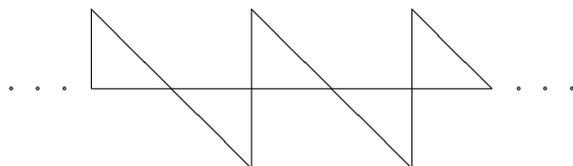


## MATH2022 Groups and vector spaces

### EXERCISES IV Cyclic groups, isomorphisms, cosets

Hand in your solutions on November 2nd

- Which of the following groups are cyclic?
  - $\mathbb{C}$  under  $+$ .
  - the set of functions from  $\mathbb{R}$  to  $\mathbb{R}$  of the form  $f(x) = x + a$  for some  $a \in \mathbb{Z}$ , under composition of functions.
  - $\mathbb{Z}_{15}^*$ .
  - $\mathbb{Z}_{14}^*$ .
  - $\mathbb{Z}_{11}^*$ .
  - The group  $G$  of isometries of the plane preserving the following frieze:



- Find which pairs of the following groups are isomorphic:
  - The dihedral group  $D_6$  of symmetries of a regular hexagon.
  - $\mathbb{Z}_6$
  - $\mathbb{Z}_{12}$
  - The group of rotations of a regular hexagon
  - The group of rotations of a regular tetrahedron.
- Find the (right) cosets for the following subgroups of the quaternion group  $Q_8$ , whose multiplication table is given in the lecture notes: (i) the subgroup  $H = \{1, -1\}$ ; (ii) the subgroup  $K = \{1, i, -1, -i\}$ .
- Find all subgroups of  $\mathbb{Z}_{20}$ . Determine the right cosets of the subgroup  $\{0, 10\}$ .
- Write out a proof of Lagrange's Theorem, using left cosets instead of right cosets. (For this you can use the equivalence relation given by  $x \sim y$  if  $x^{-1}y$  lies in the subgroup  $H$ .)
- Show that the groups of rotations of a regular tetrahedron, and of a regular hexagonal prism are non-isomorphic, non-abelian groups of order 12.
- Show that  $\mathbb{Z}_{14}$  is isomorphic to  $\mathbb{Z}_2 \times \mathbb{Z}_7$ , but that  $\mathbb{Z}_{24}$  is not isomorphic to  $\mathbb{Z}_4 \times \mathbb{Z}_6$ .
- Prove that the group of real numbers under addition is isomorphic to the group of positive real numbers under multiplication. (Hint; how can you change  $+$  into  $\times$ ?)