# Mathematical Methods in Inverse Scattering and Spectral Theory

11 - 15. September 2017, University of Leeds

Schedule of Talks

| Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|
| **10:00** Registration | **10:00** Paternain | **10:00** Guillarmou | **10:00** Levitin | **10:00** D. S. Ferreira |
| **11:00** Daude | **11:00** Oksanen | **11:00** Zelditch | **11:00** Macia | **11:00** Sincich |
| **12:00** Lunch Break | **12:00** Lunch Break | **12:00** Lunch Break | **12:00** Lunch Break | **12:00** Lunch Break |
| **13:30** Jollivet | **14:00** Rowlett | Free | **14:00** Eswarathasan | **13:00** Cekic |
| **14:30** Rogers | **15:00** Coffee Break | Afternoon | **15:00** Coffee Break | End |
| **15:30** Coffee Break | **15:30** Ruland | Discussions | **15:30** Holman | |
| **16:00** Nguyen | | | | |
| | | | **19:30** Conf. Dinner | |

# Titles & Abstracts:

**Mihajlo Cekic: The Calderon problem for connections: two perspectives**
*Abstract:* We will consider the problem of identifying a unitary connection $\nabla$ on a vector bundle, up to gauge equivalence, from the Dirichlet-to-Neumann map of the connection Laplacian $\nabla^*\nabla$. One possible approach is through the construction of special Complex Geometric Optics solutions and a further reduction of the problem to an X-ray transform. We will also consider another approach in the Yang-Mills connections setting, based on picking a special gauge in which the Yang-Mills equations become elliptic and using a unique continuation principle for elliptic systems for identification near the boundary.

**Thierry Daude: On the hidden mechanism behind non-uniqueness for the anisotropic Calderon problem with data on disjoint sets**
*Abstract:* In this talk, we shall show that there is generically non-uniqueness for the anisotropic Calderon problem at fixed frequency when the Dirichlet and Neumann data are measured on disjoint sets of the boundary. More precisely, we first show that given a smooth compact connected Riemannian manifold with boundary (M,g) of dimension higher than 3, there exist in the conformal class of g an infinite number of Riemannian metrics such that their corresponding DN maps at a fixed frequency coincide when the Dirichlet and Neumann data are measured on disjoint sets. The conformal factors that lead to these non-uniqueness results for the anisotropic Calderon problem satisfy a nonlinear elliptic PDE of Yamabe type on the original manifold (M,g) and are associated to a natural but subtle gauge invariance of the anisotropic Calderon problem with data on disjoint sets. We then construct a large class of counterexamples to uniqueness in dimension higher than 3 to the anisotropic Calderon problem at fixed frequency with data on disjoint sets "modulo this gauge invariance". This class consists in cylindrical Riemannian manifolds with boundary having two ends (meaning that the boundary has two connected components), equipped with a suitably chosen warped product metric.

**Suresh Eswarathasan: $L^2$ restriction of eigenfunctions to random Cantor-type sets**
*Abstract:* Let $(M, g)$ be a compact Riemannian surface without boundary. Consider the corresponding $L^2$-normalized Laplace-Beltrami eigenfunctions. In joint work in progress with Malabika Pramanik (U. British Columbia), I will present a result on the $L^2$ restriction of these eigenfunctions to random Cantor-type sets. This, in some sense, is complementary to the smooth submanifold $L^p$ restriction results of Burq-Grard-Tzetkov 2006 (and later work of other authors). Our method includes concentration inequalities from probability theory in addition to the analysis of singular Fourier integral operators.

**David Dos Santos Ferreira: On the linearized Caldern problem in the anisotropic setting**
*Abstract:* We are interested in the following problem which comes from the linearization of the

anisotropic Caldern inverse problem: in a compact Riemannian manifold boundary does the set of products of harmonic functions form a complete set? We study this problem in the case where the manifold is transversally anisotropic. In this setting, previous works have shown uniqueness in the nonlinear problem provided the transversal manifold was either simple or such that the geodesic ray transform was injective. Our motivation is to try to develop methods which do not appeal to the geodesic ray transform to tackle the inverse problem. The novelty is to use products of complex geometric optics solutions which concentrate on different geodesics in the transversal manifold. So far we are only able to recover singularities in the linearized problem. This is a joint work with Yaroslav Kurylev, Matti Lassas, Tony Liimatainen and Mikko Salo.

**Colin Guillarmou: TBA**
*Abstract:* TBA.

**Sean Holman: Geometrical conditions for stable inversion of the geodesic X-ray transform**
*Abstract:* This talk will cover recent progress on classification of the types of non-trapping manifolds on which the geodesic X-ray transform, which integrates functions along geodesics, possibly with a weight, can be inverted stably. There are major differences between the two and three dimensional cases arising primarily because in two dimensions there is exactly one geodesic normal to any co-vector, while in three dimensions there is a one dimensional space of such geodesics. Because of this, if there is no weight, in the two dimensional case stability fails as soon as there are conjugate points, while in three dimensions more detailed conditions must be considered such as the type of conjugate points and the so-called ?graph condition?. The talk will cover these conditions with numerical examples.

**Alexandre Jollivet: Estimates for the Steklov spectral zeta function of a planar domain and a compactness theorem**
*Abstract:* We address the question of determining a bounded smooth and simply connected planar domain $\Omega$ from the spectrum of its Dirichlet-to-Neumann operator (Steklov spectrum). We state two analog formulations of this problem. Then we prove that, for a fixed real $s$ satisfying $|s| > 1$ and fixed length $L(\partial\Omega)$ of the boundary curve, the zeta function $\zeta_\Omega(s)$ of the Steklov spectrum reaches its minimum when $\Omega$ is a disk. This result is obtained by studying the difference $\zeta_\Omega(s) - 2(\frac{L(\partial\Omega)}{2\pi})^s\zeta_R(s)$, where $\zeta_R$ stands for the classical Riemann zeta function. Next we provide some estimates from below for the Steklov zeta invariants [Malkovich-Sharafutdinov, 2015] which are the values of the zeta function $\zeta_\Omega$ at even negative integers. The estimates allow us to prove the compactness, with respect to the $C^\infty$ topology, of a Steklov isospectral family of planar domains. This generalizes a precompactness statement with respect to the $H^s$-topology with $s < \frac{5}{2}$ given in [Edward, 1993]. These are joint works with Vladimir Sharafutdinov (Novosibirsk State University & Sobolev Institute of Mathematics).

**Michael Levitin: Sloshing, Steklov and corners**
*Abstract:* I will speak about sharp eigenvalue asymptotics in the sloshing problem and the Steklov problem on domains with corners. This is a joint work with L Parnovski, I Polterovich, and D Sher.

**Fabricio Macia Lang: Concentration of quasimodes for perturbed integrable systems**
*Abstract:*We study concentration and non-concentration phenomena for quasimodes and solutions to the time-dependent Schrdinger evolution for perturbations of completely integrable systems. We show the existence of an hierarchy of time scales for which the perturbation starts affecting the dynamics of concentration of solutions. As an application, we present necessary conditions on the structure of Quantum Limits. This is based on joint work with V. Arnaiz (Madrid) and G. Rivire (Lille).

**Lauri Oksanen: The light ray transform and applications**
*Abstract:* The light ray transform of a function on a Lorentzian manifold is the map giving the integrals of the function over each light ray (that is, null geodesic). We show that the light ray transform is invertible when the Lorentzian manifold is the product of the real line and a Riemannian manifold on which the geodesic ray transform is invertible. Moreover, we discuss the microlocal structure of the normal operator associated to the light ray transform. We give applications to hyperbolic inverse boundary value problems and to the study of Cosmic Microwave Background measurements. The talk is based on joint works with Yavar Kian and with Matti Lassas, Plamen Stefanov and Gunther Uhlmann.

**Gabriel Paternain: Lens rigidity for a particle in a Yang-Mills field**
*Abstract:* We consider the motion of a classical colored spinless particle under the influence of an external Yang-Mills potential A on a compact manifold with boundary of dimension $\geq 3$. We show that under suitable convexity assumptions, we can recover the potential A, up to gauge transformations, from the lens data of the system, namely, scattering data plus travel times between boundary points. This joint work with Gunther Uhlmann and Hanming Zhou.

**Keith Rogers: The Calderon inverse problem with Lipschitz conductivities**
*Abstract:* We will consider the Calderon inverse problem in three and more dimensions. That is the problem of recovering the conductivity of a body from electrical measurements on the boundary. After converting to the associated inverse scattering problem, and recalling the uniqueness argument of Haberman and Tataru, I will provide a reconstruction formula that works for Lipschitz conductivities sufficiently close to one. A different argument was previously provided by Garca and Zhang. I will then present the difficulties in removing the closeness condition. This is joint work with Pedro Caro and Jorge Tejero.

**Julie Rowlett: Hearing the shape of a trapezoid drum**

*Abstract:* I will discuss joint work with H. Hezari and Z. Lu concerning isospectrality of trapezoidal domains. In 1966, M. Kac popularized the isospectral problem for planar domains in his paper, "Can one hear the shape of a drum?" While the answer to this question was demonstrated to be "no" by C. Gordon, D. Webb & S. Wolpert in 1991, if one restricts to certain specific types of domains, the answer can be "yes." For example, C. Durso proved in 1988 that one can hear the shape of a triangular domain, in the sense that if two triangular domains are isospectral, then they are congruent. Her proof involved the wave trace; a simpler proof involving only the heat trace was obtained by D. Grieser and S. Maronna in 2012. We consider isospectral trapezoidal domains, with either Dirichlet or Neumann boundary condition. We prove that with either boundary condition, if two trapezoidal domains are isospectral, then they are congruent. Interestingly, the proofs in these two cases are different, because in a certain sense that we shall make precise, the wave trace in the Neumann case is "more singular" than in the Dirichlet case.

**Angkana Ruland: Stability for the fractional Calderon problem**
*Abstract:* In this talk I discuss stability properties of the fractional Calderon problem in the presence of rough potentials. Based on a careful analysis of propagation of smallness estimates and duality arguments, I derive quantitative Runge approximation properties for the fractional Laplacian in the first part of the talk. In the second part of the talk, these are then used to obtain logarithmic stability properties for the fractional Calderon problem. This is based on joint work with Mikko Salo.

**Eva Sincich: Lipschitz stability for a piecewise linear Schrödinger potential from local Cauchy data**
*Abstract:* We consider the inverse boundary value problem for the stationary Schrödinger equation consisting in the determination of the potential q from local Cauchy data. In view of the well-known exponential ill-posedness of this problem we introduce the a-priori assumption on the unknown coefficient q of being piecewise linear on a given domain partition and we obtain a conditional Lipschitz stability result. No sign, nor spectrum condition on q is assumed, hence the present analysis encompasses the reduced wave equation at fixed frequency. This inverse problem arises, for example, in reflection seismology and inverse obstacle scattering problems for acoustic and electromagnetic waves. This is based on a joint work with G. Alessandrini, M. V. de Hoop and R. Gaburro.

**Steve Zelditch**
*Abstract:* Many years ago, Toth and I proved that for real analytic plane domains, the number of zeros of an eigenfunction on a "good" real analytic curve was bounded by the frequency. 'Good' is a kind of non-degeneracy condition resembling a Carleman lower bound on the curve. My talk first gives a generalization to all dimensions: in any dimension and for any real analytic metric, the number of zeros of the restriction of the eigenfunction to a 'good curve' is bounded by the frequency. We also prove the result for good hypersurfaces. Moreover we give a robust criterion for a hypersurface to be 'good' in all dimensions. Roughly speaking it is good if it is asymmetric with respect to geodesics and if the flowout of the unit sphere bundle along the curve fills out the unit

sphere bundle in measure. Joint work with J. Toth.