Fredholm Theory of Non-Elliptic Operators

3 - 7. June 2019, University of Leeds

Schedule of Talks

Monday	Tuesday	Wednesday	Thursday	Friday
		9:00 Ruzhansky	**9:00** Zelditch	**9:00** Melrose
10:00 Registration	**10:00** Wrochna	**10:00** Schrohe	**10:00** Fischer	**10:00** Bahns
11:00 Guillarmou	**11:00** Weich	**11:00** Coffee break	**11:00** Coffee break	**11:00** Coffee break
12:00 Lunch break	**12:00** Lunch break	**11:30** Häfner	**11:30** Gerard	**11:30** Singer
13:30 Bär	**14:00** Balehowsky	**12:30** Lunch break	**12:30** Lunch break	**12:30** Lunch break & End
14:30 Bismut	**15:00** Coffee Break	Free Afternoon & Discussions	**14:00** Hintz	
15:30 Coffee break & Discussions in reading room 9.31	**15:30** Shargorodsky		**15:00** Coffee break	
			15:30 Ludwig	
			19:00 Conf. Dinner	

All talks take place in the **lecture theatre LT 11 in the Roger Stevens building** (10.11).
Talks are 50 minutes and there is a 10 min break between the talks.
Coffee-breaks take place in the Level 9 Foyer in the main mathematics building.

Titles & Abstracts:

Dorothea Bahns: Construction of a Haag Kastler net of von Neumann algebras

Abstract: Based on the functional formalism for perturbative Algebraic Quantum Field Theory, the Haag-Kastler net of local von Neumann algebras is constructed in the ultraviolet finite regime of the sine-Gordon model. This gives an example of a theory satisfying the Haag-Kastler axioms. The approach does not need an auxiliary mass nor the detour of Euclidean signature.
(This is joint work with Kasia Rejzner and Klaus Fredenhagen).

Tracey Balehowski: Recovering a Riemannian metric from least-area data

Abstract: In this talk, we address the following question: Given any simple closed curve γ on the boundary of a Riemannian 3-manifold (M, g), suppose the area of the least-area surfaces bounded by γ are known. From this data may we uniquely recover g? In several settings, we show the the answer is yes. In fact, we prove both global and local uniqueness results given least-area data for a much smaller class of curves on the boundary. We demonstrate uniqueness for g by reformulating parts of the problem as a 2-dimensional inverse problem on an area-minimizing surface. In particular, we relate our least-area information to knowledge of the Dirichlet-to-Neumann map for the stability operator on a minimal surface. Broadly speaking, the question we address is a dimension 2 version of the classical boundary rigidity problem for simply connected, Riemannian 3-manifolds with boundary, in which one seeks to determine g given the distance between any two points on the boundary. We will also briefly review this problem of boundary rigidity as it relates to aspects of our question of recovering g from knowledge of areas.
(This is joint work with S. Alexakis and A. Nachman).

Christian Bär: Boundary value problems for the Dirac operator on Riemannian and Lorentzian manifolds

Abstract: Boundary value problems for the Dirac operator on a Riemannian manifolds are rather well understood. In particular, one has a general description of admissible boundary conditions. The Lorentzian case has been studied only recently and it turns out that there are similarities but also fundamental differences to the Riemannian case. I will describe both situations and contrast them.
(This is joint work with L. Bandara, W. Ballmann, S. Hannes and A. Strohmaier).

Jean Michel Bismut: A Riemann-Roch theorem in Bott-Chern cohomology

Abstract: I will describe a geometric problem on families of elliptic operators, which is solved via a deformation to a family of non self-adjoint Fredholm operators. Let $p : M \to S$ be a proper holomorphic projection of complex manifolds. Let F be a holomorphic vector bundle on M. We assume that the $H^{(0;p)}(X_s; F|_{X_s})$ have locally constant dimension. They are the fibers of a holomorphic vector bundle on S. The problem we will address is the computation of characteristic classes associated with the above vector bundle in a refinement of the ordinary de Rham cohomology

of S, its Bott-Chern cohomology. When M is not Kähler, none of the existing techniques to prove such a result using the fiberwise Dolbeault Laplacians can be used. The solution is obtained via a proper deformation of the corresponding Dolbeault Laplacians to a family of hypoelliptic Laplacians, for which the corresponding result can be proved. This deformation is made to destroy the geometric obstructions which exist in the elliptic theory.

Veronique Fischer: Towards subelliptic QE?

Abstract: This talk will discuss the recent advances on pseudo-differential theory in sub-Riemannian or sub-elliptic settings, in particular regarding index theorems and in a different direction tools to study quantum ergodicity.

Christian Gerard: The Feynman propagator for Klein-Gordon equations on curved spacetimes

Abstract: Feynman propagators are at the heart of Quantum Field Theory on curved spacetimes. Mathematically they are distinguished inverses of Klein-Gordon operators on curved spacetimes, characterized by the wave front set of their distributional kernels. In particular each physically reasonable state of the quantized Klein-Gordon field defines a Feynman propagator, but on general spacetimes, there is no canonical and unique choice of it. We will explain some recent work with Michal Wrochna, where we define a canonical Feynman propagator on spacetimes which are asymptotically Minkowskian at time and space infinity.

Dietrich Häfner: Linear stability of slowly rotating Kerr spacetimes I: overview

Abstract: In joint work with Peter Hintz and Andras Vasy, we study the asymptotic behavior of linearized gravitational perturbations of Schwarzschild and slowly rotating Kerr black hole spacetimes. We show that solutions of the linearized Einstein equation decay at an inverse polynomial rate to a stationary solution (given by an infinitesimal variation of the mass and angular momentum of the black hole), plus a pure gauge term. In this talk, I will describe the geometric background and the analytic setup of our result, including a discussion of gauge fixing. Our proof is based on a precise description of the resolvent of an associated wave equation on symmetric 2-tensors near zero energy; further details will be given in the talk by Peter Hintz.

Peter Hintz: Linear stability of slowly rotating Kerr spacetimes II: low frequency analysis

Abstract: This is a continuation of the talk by Dietrich Häfner on our joint work with Andras Vasy on the asymptotic behavior of linearized gravitational perturbations of slowly rotating Kerr black hole spacetimes. I will explain aspects of the low frequency analysis of the spectral family in this context, with a focus on the description of the space of (generalized) zero energy bound states, the role of constraint damping, and the underlying robust low energy Fredholm analysis.

Ursula Ludwig: An Extension of a Theorem by Cheeger and Müller to Spaces with Isolated Conical Singularities

Abstract: An important comparison theorem in global analysis is the comparison of analytic and topological torsion for smooth compact manifolds equipped with a unitary at vector bundle. It has been conjectured by Ray and Singer and has been proved independently by Cheeger and Müller in the 70ies. The aim of this talk is to present an extension of the Cheeger-Müller theorem to spaces with isolated conical singularities by following the Bismut-Zhang philosophy.

Richard B. Melrose: Operators on Loop Space

Abstract: I will outline some of the steps, analytic, topological, algebraic and geometric, needed to define the Dirac-Ramond operator on the loop space of a manifold with (bundle having) a string structure. Obstacles remain, as I will indicate, to the description of index theory in this infinite-dimensional setting.

Michael Singer: Dirac operators coupled to Dirac monopoles and the geometry of configuration spaces

Abstract: We consider a family D_s of Dirac operators coupled to a fixed Dirac monopole with singularities at points $p_1, \ldots, p_n$ in R^3. Here s is a real positive parameter. Using a suitable Fredholm set-up, we prove that the L^2 null-space N_s of D_s is of dimension n for all s. Further, by considering the limiting behaviour for small and large s, we obtain on the one hand a natural isomorphism of N_s with the space of complex polynomials of degree $n-1$ and on the other a canonical basis of N_s. Since this basis depends upon the points $p_1, \ldots, p_n$, this gives a natural map from the configuration space of points into the flag manifold $U(n)/T^n$. Such a natural map was constructed by Atiyah and Bielawski our discussion gives an interpretation of their map in terms of Dirac monopoles.

Elmar Schrohe: Degenerate Elliptic Boundary Value Problems with Non-smooth Coefficients

Abstract: On a manifold of bounded geometry with boundary we consider a uniformly strongly elliptic second order operator A that locally takes the form

$$A = \sum_{j,k} a_{jk} \partial_j \partial_k + \sum_j b_j \partial_j + c$$

together with a degenerate boundary operator T of the form

$$T = \varphi_0 \gamma_0 + \varphi_1 \gamma_1$$

where γ_0 and γ_1 denote the evaluation of a function and its exterior normal derivative, respectively, at the boundary, and φ_0, φ_1 are smooth functions on the boundary with $\varphi_0 > 0, \varphi_1 >\geq$ and $\varphi_0 + \varphi_1 > 0$. Unless either $\varphi_0 = 0$ or $\varphi_1 = 0$ this problem is not elliptic in the sense of Lopatinskij and Shapiro. We show that the realization A_T of A in $L^p(\Omega)$ has a bounded H^∞-calculus whenever the a_{jk} are Hölder continuous and b_j as well as c are L^1. For the proof we first treat the operator with smooth coefficients on R_+^n. Here we rely on an extension of Boutet de Monvel's calculus to operator-valued symbols of Hörmander type $(1, \delta)$. We then use H^∞-perturbation techniques in order to treat the nonsmooth case. As an application we study the porous medium equation. (Joint work with Thorben Krietenstein, Hannover).

Eugene Shargarodsky: Non-classical elliptic boundary value problems arising in the theory of Markov processes

Abstract: The aim of the talk is to present results of an ongoing joint project with Tony Hill concerned with non-classical boundary value problems for elliptic pseudodifferential operators that arise in the study of Markov processes with jumps in the case where the state space has a boundary. Particular attention will be paid to a model Wiener-Hopf plus Mellin operator, which is related to the generator of a symmetric stable Levy process on a half-line.

Tobias Weich: Ruelle-Pollicott resonances for manifolds with cusps

Abstract: In this talk I want to explain how the notion of Ruelle-Pollicott resonances can be extended to manifolds with cusps. More prescisely I will consider the geodesic flow on a manifold of strict negative sectional curvature that decomposes into a compact part and a finite number of hyperbolic cusps. I will then discuss how the resolvent of the geodesic flow vector field can be continued meromorphically using microlocal analysis.
(This is joint work with Yannick Bonthonneau).

Michal Wrochna: Global inverses of the wave operator in Quantum Field Theory

Abstract: In Quantum Field Theory, fundamental difficulties arise from the need of finding distinguished inverses of the wave, Klein-Gordon or Dirac operator on a Lorentzian manifold. In this talk, I will give a brief introduction to QFT on curved manifolds, focusing first on the role of (approximate) inverses, on the construction of quantum fields and on their conjectured relationships with geometry. After reviewing results known in the Riemannian case (including recent work by N.V. Dang), I will give an overview of some recent developments in the physical Lorentzian case. Part of the talk based on joint works with C. Gerard and with A. Vasy.

Steve Zelditch: Observability estimate from a hypersurface

Abstract: The basic question is: find necessary and sufficient conditions that the restriction of a sequence of eigenfunctions to a hypersurface H tends to zero in $L^2(H)$. In particular, if H is a curve on a hyperbolic surface, what does the Dyatlov-Jin observability estimate imply about the sequence?