What Makes a Computation Unconventional?
or, there is no such thing as Non-Turing Computation

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Turing’s standard model of computation, and its physical counterpart, has given rise to a powerful paradigm. There are assumptions underlying the paradigm which constrain our thinking about the realities of computing, not least when we doubt the paradigm’s adequacy.

There are assumptions concerning the logical structure of computation, and the character of its reliance on the data it feeds on. There is a corresponding complacency spanning theoretical – but not experimental – thinking about the complexity of information, and its mathematics. We point to ways in which classical computability can clarify the nature of apparently unconventional computation. At the same time, we seek to expose the devices used in both theory and practice to try and extend the scope of the standard model. This involves a drawing together of different approaches, in a way that validates the intuitions of those who question the standard model, while providing them with a unifying vision of diverse routes “beyond the Turing barrier”.

The results of such an analysis are radical in their consequences, and break the mould in a way that has not been possible previously. The aim is not to question, invalidate or supplant the richness of contemporary thinking about computation. A modern computer is not just a universal Turing machine. But the understanding the model brought us was basic to the building of today’s digital age. It gave us computability, an empowering insight, and computing with consciousness. What is there fundamental that unconventional computation directs us to? What is it makes a computation unconventional? And having fixed on a plausible answer to this question, we ask: To what extent can the explanatory power of the mathematics clarify key issues relating to emergence, basic physics, and the supervenience of mentality on its material host?
1 METHOD OVER MATTER

There is a huge literature concerning computability. It has grown beyond what anyone might have anticipated back in the 1930s. The subject has taken on a life of its own, the context has spread across disciplinary boundaries in a startling way and ideas are increasingly hard to categorise and evaluate within traditional structures. During the 2012 centenary of Alan Turing’s birth, we were reminded that some of the most important early contributions to the computing revolution came from people who thought deeply about the way the world computes, while gaining strength from their independence of what other people had done. Thank you, Turing. Relieved of the burden of the taxonomy of who said what and when, let us start by mapping out some of the main features of the standard model of computation. Or rather, the main features of how a starting student in computability might unwrap the model.

The most basic feature, a feature not just of Turing’s 1936 Turing machine but of all the equivalent models, is its disembodiment. We might have been told at some point that it was devised as a disemboding model of machine computation. Not so, of course. 2012 has made everyone aware of the very specific physicality of the computing situation that Turing was modelling, the predominantly women ‘computers’ following instructions. One of the special strengths of the Turing model is its close relationship with physical computation, via a very specific deconstruction of a typical computational context. We have but a confused idea of what a machine might be. We have a firmer grip on what a ‘computer’ following instructions might be doing, using well-defined workspace, tools and conventions. The underlying physicality may be highly complex. But such things as the human computer’s aches and pains, her feelings of hunger or boredom, are factored out of the process. We extract emergent features of the material context which are far from disembodings of the computation, but which give us a model which we may re-embody in quite different contexts, and whose mathematical properties can be investigated with a realistic hope of relevance to a wide spectrum of controlled situations.

This provides a green light to those who would turn such a superficial take on physical computation back on its host. In the absence of a corresponding deconstruction of more complex physicality – and ignoring the fact that even more ad-hoc descriptions of particle physics, life, cosmology, human mentality etc. are incomplete – the temptation is to turn the Turing model into metaphor, and then into extended model, in ways we can only argue about. The disembodiment implicit in the standard model is not so simple. It has a character which we should not ignore. Surfing the compu-
tational world is fun, but the underlying complexity may still surprise.

We have dwelled on the basic particularity and oddness of the standard model. It is relevant to what we expect of other aspects of the dominant paradigm. Before Turing and logicians like Emil Post, Kurt Gödel, Alonzo Church, and Stephen Kleene came on the scene, an important input to a computation was the computing machine itself. Physically the machine embodied a weighty piece of data. The logician’s overview provided an extraction of its essentials in the form of a simple code – a natural number, a finite binary string, or other similar mathematical object. This provided two hugely important features of the standard model, and the modern computational world.

Firstly, however we structure our machines, their descriptions can be converted into data used by machines based on a different logical analysis, enabling the construction of algorithms for converting ‘programs’ within one framework into one fitting with another. Much activity of the early investigators involved devising such algorithms (say, for converting a description of a $\lambda$-computable function into one for an equivalent Turing computable function). The natural conclusion was that Turing machines could ‘compute anything’.

Secondly, having trivialised the hardware, the power of the computing paradigm lay in the programming. And with a model which turned machines into data, it was a short step to building a machine which could mine machines for different purposes out of whatever data you gave it – and having decoded the program from the data, could implement it. This was the origin of the so-called ‘universal Turing machine’. By a simple mathematical sleight of hand, one had moved machine from physical world to the realm of pure thought. Well, not quite, as Charles Babbage found out in the process of getting his Analytical Engine built prior to 1871. Of course, Babbage’s machine, like others pre-dating the 1948 Manchester Small-Scale Experimental Machine (SSEM), or ‘Manchester Baby’, was not universal machine in the sense of a modern stored-program computer.

Today’s computers are a true embodiment of Turing’s universal machine in that they enable programs to be combined and edited in increasingly creative ways, without the need for any rebuilding of the programmer-computer physical interface. Nowadays, the program, once input, becomes part of the computer, to be stored, adapted or discarded by the programmer without any grappling with punched cards or realigning of wired connections or switches. Early designs of Babbage, Konrad Zuse and others are ‘Turing complete’, but lack the vital stored-program feature. This important ingredient of Turing’s 1936 logical analysis was incorporated by John von
Neumann in his June 1945 EDVAC report, and also features, much less influentially, in Turing’s report on the ACE of later that same year.

Universality; the transposability of computational activity from one computing platform to another; the supplanting of the physical by the logical; the redundancy of information beyond the type 1 or type 2 mathematical level — these are familiar aspects of an overarching computational paradigm. The underlying assumptions have served us well, and moulded our thinking about the wider context. One can recognise it in early approaches to artificial intelligence. In the philosophy of mind one has various functionalist viewpoints, with Hilary Putnam explicitly drawing on the universal machine metaphor in his seminal 1960s writings on the topic. Again, in computer science one has the allied notion of ‘virtual machine’ quite validly useful in both practical and theoretical contexts. One observes the paradigm in the drive to reduce social interaction and development to the algorithmic, setting complex interactive processes within simple rule-based game structures. The feedback between the emergent and the algorithmic, to which we return below, does not fit well with ‘corporate thinking’, with its drive for strategic certainty.

In computer science and mathematics the paradigm can be detected in sophisticated approaches to the logic of computation, focused on the value of frameworks transferable not just between specific contexts, but between different disciplines. Categorical methods have been productive in the computational context, where according to Samson Abramsky “in the work on concurrent processes, the behaviour is the object of interest.

2 PROCESS AND EMBODIMENT

History brings its own reminders that computers are not ‘just’ universal Turing machines. Moving the model from the human ‘computers’ platform to a more efficient and cost-effective electronic platform was highly non-trivial. Apart from this, the re-embodiment of computing brought us closer to the main point of Turing’s 1936 paper — a proof that there are interesting questions beyond the reach of algorithms. In retrospect, one can rephrase this as “computers are stupid”, and go on to ask if a 14 billion year old universe is subject to similar limitations. Is the mathematics of Turing’s simple diagonalisation of the computable reals unembodiable?

The sort of problems Turing and Alonzo Church showed to be unsolvable by a computer were very natural in an everyday sense. From Turing we know that if $U$ is a universal Turing machine then there is no computer can tell
us, for an arbitrary input $x$, whether $U$ will ever produce an output from $x$. This is the ‘unsolvability of the Halting Problem’ for $U$, with the set of numbers $x$ on which $U$ halts called the Halting Set for $U$. Remembering that $x$ can code a program, this gives us an indication of why computer program checking is such a tough problem. The process tends to be experimental, with a new piece of software requiring a sequence of updates to fix various bugs.

Even closer to home is ‘Church’s Theorem’ — actually Turing proved it too, it is just the negative solution to Hilbert’s *Entscheidungsproblem*. This says that if you have a sentence in everyday language (as formalised in first-order logic), there is no computer that can tell us of any such sentence whether it is logically valid or not. To many this is quite informative and counter-intuitive.

One can extract from each of these problems a binary expression for a real number $r$. Say $r = 0 \cdot r_0r_1 \ldots$, with each $r_i = 0$ or $1$, where $r_i = 1$ exactly when $U$ successfully computes on input $x = i$. $U$ can be thought to ‘compute’ $r$ in the sense that this number is uniquely decided by the actions of $U$ in computing on $0, 1, 2, \ldots$ successively. $r$ is a very real *feature* of the real world in which $U$ lives and operates. However, the level of abstraction of $r$ means that even though we can ‘see’ $U$ computing, we cannot ‘see’ $r$ at all well. If we could see $r$ we would be moved to allow that it it is ‘computed’ in some sense. Of course, $r$ does not fit into the classical paradigm, since $r$ is not available as an input to further computation by $U$. Not only can we not see $r$, nor can $U$. And this is not just due to the incomputability. $r$ as a mathematical object is of higher *type* than the natural numbers which $U$ usually accepts as an input. Anyway, in the absence of an embodied presence, $r$ is not considered a computed outcome.

Mathematically, $r$ is *definable* from $U$, but the existential quantifier needed to define it puts $r$ on the other side of an unembodied chasm. The question arises: Can this chasm be crossed given the right material conditions?

There is an obvious counterpart of this elevation of type, and Turing’s proof of a resulting incomputability. The non-locality of the view amounts to a logical interactivity between computations. The observing process provides the connectivity, with us a player in the physical environment. We are no longer in the presence of an individual computation, it is an interactive *process* at work, with what we will subsequently recognise as an *emergent* incomputable real $r$. We will come to regard *emergent* as real world analogue of ‘definable’. Emergence plays an important role in many dynamical systems, such as weather systems, large scale social interactions, the internet,
biology, creative thinking, and turbulent environments of many kinds.

Definability is commonly ignored, or regarded as a logician’s playground, with important instantiations in the wider mathematical context. A useful ‘missing link’ is the fractal, with both precise mathematical description and a visual presence, often enhanced via computer simulation.

In a formal sense, the Halting Problem is in the same world as the Mandelbrot set, for instance. We have gone up another level of the type structure, but there is an in-principle connection. We have a simple computable rule hosted by the complex numbers. Based on this there is a two-quantifier definition of the members of the Mandelbrot set, which with a little manipulation can be reduced to a one-quantifier expression for the purpose of the well-known computer simulation. The computability or otherwise of the Mandelbrot set is still an open problem. But unlike our incomputable Halting Set, the Mandelbrot set comes beautifully and interestingly embodied, with quite visual counterparts to the suspected incomputability. There it is on our computer screen. And we can delve as deeply as we like into this fascinatingly surprising type-2 object. The reason for this is that we are sampling this set of complex numbers. The computer screen image involves a trick reduction of type. Turing himself was familiar with the usefulness of statistical sampling for reducing complex information to something computationally approachable. It is not a purely ad hoc methodology. It is a way of recognising the higher type computability enabled by a definition, or by some real world process to whose computational underpinnings we are not privy.

Moving beyond our mathematical comfort zone, we can observe many everyday examples of emergence as instances of objects definable in a real context. We see apparently chaotic environments involving generation of informational complexity via simple rules with a computational character. And we further observe the accompanying, often surprising, emergence of new regularities — such as those of Robert Shaw’s dripping taps — entropic resting points, often at most observable via the sort of selective sampling which made visible the embodied Mandelbrot set.

The embodied computation of higher type objects is not in itself a challenge to the classical model. But its character does mimm with intuitions concerning unconventionality of computation. And the parallel with the established incomputable \( r \) and its mathematical context certainly rings the alarm bells.
3 EMERGENCE AND DEFINABILITY

All around us we see a world exhibiting algorithmic content accompanied by hierarchical structure not easily explainable in terms of the familiar underlying rules. Our everyday lives are built around what appears to be a computable environment, but nature continually surprises us. Much of that surprise is attached to natural form which does appear to be part of a universe which ‘knows what it is doing’, and it is this we think of as ‘emergent’.

The importance of getting a mathematical grip on this omnipresent phenomenon — in evidence from ‘strange attractors’ to human creativity, and from the origins of life to large-scale cosmic structure — is illustrated by the history of ‘British Emergentism’, and its heyday in the 1920s. One of the leaders of the movement was the Cambridge philosopher C. D. Broad. Here he is in 1925, attempting an explanation of what emergence is, while pointing to illustrative examples:

\[ \ldots \text{the characteristic behaviour of the whole} \ldots \text{could not, even in theory, be deduced from the most complete knowledge of the behaviour of its components} \ldots \text{This} \ldots \text{is what I understand by the ‘Theory of Emergence’. I cannot give a conclusive example of it, since it is a matter of controversy whether it actually applies to anything} \ldots \text{I will merely remark that, so far as I know at present, the characteristic behaviour of Common Salt cannot be deduced from the most complete knowledge of the properties of Sodium in isolation; or of Chlorine in isolation; or of other compounds of Sodium,} \ldots \]

Dramatic scientific developments were in progress around this time. 1925 saw the key elements of the new quantum mechanics put in place by Werner Heisenberg and Erwin Schrödinger, and by the 5th Solvay conference in 1927 quantum theory was revolutionising the foundations of chemistry. The mystery was stripped from the examples from chemistry of Broad and others.²

For Stuart Kauffman³ emergence is not just an example of unconventional computation, it calls into question basic assumptions about the computational content of causality and the deterministic character of the universe:

We are beyond reductionism: life, agency, meaning, value, and even consciousness and morality almost certainly arose naturally, and the evolution of the biosphere, economy, and human culture are stunningly creative often in ways that cannot be foretold, indeed in ways that appear to be partially lawless. The latter challenge to current science is radical. It runs starkly counter to almost four hundred years of belief that natural laws will be sufficient to explain what is real anywhere in the universe, a view I have called the Galilean spell. The new view of emergence and ceaseless creativity partially beyond natural law is a truly new scientific worldview in which science itself has limits.

Such claims are counterbalanced by words of caution from Ronald Arkin:\footnote{Ronald C. Arkin, \textit{Behaviour-Based Robotics}, MIT Press, 1998, p.105.}

Emergence is often invoked in an almost mystical sense regarding the capabilities of behavior-based systems. Emergent behavior implies a holistic capability where the sum is considerably greater than its parts. It is true that what occurs in a behavior-based system is often a surprise to the system’s designer, but does the surprise come because of a shortcoming of the analysis of the constituent behavioral building blocks and their coordination, or because of something else?

In the face of historic confusions, and radical contemporary speculations, the clarifying role of mathematics is urgently needed. This is not to brush aside the more detailed proposals of Kauffman and others. The aim is to place them in a more foundational framework.

To this end, one needs more than the codifying of current ‘best observational practice’ represented by the Test of Emergence of Ronald, Sipper and Capcarrère in \textit{Design, observation, surprise! A test of emergence} (Artificial Life, \textbf{5} (1999), 225–239). Here is a summary of their qualifying criteria:

1) **Design**: The system has been constructed by the designer, by describing local elementary interactions between components (e.g., artificial creatures and elements of the environment) in a language $\mathcal{L}_1$.

2) **Observation**: The observer is fully aware of the design, but describes global behaviors and properties of the running system, over a period of time, using a language $\mathcal{L}_2$. 
3) **Surprise**: The language of design $\mathcal{L}_1$ and the language of observation $\mathcal{L}_2$ are distinct, and the causal link between the elementary interactions programmed in $\mathcal{L}_1$ and the behaviors observed in $\mathcal{L}_2$ is non-obvious to the observer – who therefore experiences surprise. In other words, there is a cognitive dissonance between the observer’s mental image of the system’s design stated in $\mathcal{L}_1$ and his contemporaneous observation of the system’s behavior stated in $\mathcal{L}_2$.

Might this serve as a test for unconventional computation? Unconventionality certainly requires some obstacle to reduction to basic algorithmic structure. And it is hard to design a computational device which has no underpinning of classical ingredients. On the other hand, there are potentially incomputable processes in nature for which 1) or 2) fail. Can a foundational approach make computational sense of the outcome of a quantum measurement leading to a collapse of the wave function?

A nice aspect of the above test is its differentiation between ‘designer’ and ‘observer’ languages. This is a feature of the fragmentary nature of science, where it is common to view, say, biology as emergent from an underlying quantum mechanical base, with its own emergent rules and language, non-reducible to the quantum level on which it depends. In the case of the Halting Set for a universal Turing machine, $\mathcal{L}_2$ is distinguished by the addition of quantification.

Alan Turing recognised something computationally interesting in emergence when he investigated the mathematics of morphogenesis. In the early 1950s Turing wrote his groundbreaking paper on *The chemical basis of Morphogenesis*, in which he proposed a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis in a range of cases. He even ran computer programs on the early Manchester Mark 1 computer (a more powerful successor to the ‘Baby’) with the aim of verifying his reaction-diffusion ‘design’ underlying such emergent patterns as the familiar black and white dappling on a Holstein dairy cow.

What is specially interesting about this work is how it related the powerful descriptive framework of differential equations to emergent form in nature, so exhibiting a connection between the mathematics of higher type objects and apparent emergence. It is hard to claim computational unconventionality on this basis – the solutions to Turing’s equations tended to be computable – but then mathematics provides us with little means of identifying real world incomputability. Reducing the Halting Problem to an elusive solution to a non-linear differential equation is not very likely. On the other hand, Marian Pour-el and Ian Richards had some success designing
‘A computable ordinary differential equation which possesses no computable solution’\(^5\)

To summarise: Turing provided examples of descriptions of emergent phenomena, whereby one might characterise the emergence as an expression of a higher type computation. And this fits well with the Ronald-Sipper-Capcarrère test for emergence, via the provision of each of design, observation and surprise. With the latter mathematically traceable back to the type-climbing and concomitant potential incomputability of the emergent form.

Is it pure serendipity the discovery that some phenomena can be described in terms of material context? There is a strong intuition that form in the universe arises for a reason. Scientifically this intuition takes the form of an expectation of finding descriptions of phenomena in terms of basic laws of nature. An echo of such an expectation be traced back to Gottfried Leibniz’s 1714 description\(^6\) of his ‘principle of sufficient reason’:

\[
\ldots\text{there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases.}
\]

The intuition that natural phenomena not only generate descriptions, but arise and derive from them, connects with a useful abstraction associated with Alfred Tarski, and growing out of his 1930s work on the notion of truth for formal languages. Mathematical definability, or more generally invariance under automorphisms of an appropriate structure, provides an effective organiser of the relative ontology of relations over a structure.

Definability is a basic notion which deserves to be better known in the mathematical world, and in the wider scientific community. It’s relevance to physics has been long recognised. Hans Reichenbach worked to axiomatise Einstein’s relativity in the 1920s, a project carried forward in relation to general relativity today by the Budapest group of István Németi and Hajnal Andréka. This extension of the fundamental mathematics enables us to deal with a wider range of phenomena, taking us beyond the classical computational model. It gives precision to our experience of emergence as a potentially trans-algorithmic determinant of phenomena.

The overarching aim now is to describe global relations in range of contexts in terms of local structure, so capturing the emergence of large scale

\(^5\)In: Annals of Mathematical Logic Volume 17, November 1979, Pages 6190.
\(^6\)See The Monadology, sections 31, 32.
formations. And mathematically to formalise such descriptions as definability, or as invariance over basic computational structure. Although Stephen Kleene provided formal content to the notion of higher type computation via a series of papers spanning over 30 years (1959–1991), the physical relevance of his take on the topic needs to be clarified. A forthcoming book on “Computability At Higher Types” by John Longley and Dag Normann is eagerly anticipated. The intuition is that computational unconventionality certainly entails higher type computation, with a correspondingly enhanced respect for embodied information. There is some understanding of the algorithmic content of descriptions. But so far we have merely scratched the surface.

4 PHYSICS AND DEFINABILITY

When a Nobel Prize winner in Physics is quoted as saying\(^7\):

> The state of physics today is like it was when we were mystified by radioactivity . . . They were missing something absolutely fundamental. We are missing perhaps something as profound as they were back then.

people take notice. And this from 2004 winner David Gross did cause something of a stir.

This section is in the nature of a road test for the conceptual framework we have been building up around the notion of unconventionality of a computation. We briefly outline various gaps in the ‘standard model’ of physics and point to the how a more basic viewpoint can help. The discussion will consist of a brief commentary centred around some revealing quotations from physicists themselves.

We start with Einstein himself complaining about the resort to ad hoc elements of physical theories:

> . . . I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature . . . nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) . . .

\(^7\)David Gross, quoted in New Scientist, Dec. 10 2005, “Nobel Laureate Admits String Theory Is In Trouble”. 

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Notice that this is not just an exhortation to physicists to look for a better theory. It is an expression of faith in the fact that a theory which successfully defines the observable universe should itself be determined by the universe. That is, what we observe is there because the universe is 'self-organising' itself, as one would expect of an emergent system with sufficient invariance of its structure to exhibit a high degree of mathematical rigidity. An interesting question is the extent to which constants of the model which make it work, but which are not measurable, are actually defined. In general, one can interpret the necessity of certain values of the constants to make the model work as a sort of invariance. What we suspect of invariance is an elusiveness of algorithmic infrastructure to the relationship between the local and global which makes it possible that aspects of reality are dependent on basic information in a way that is impossible for us to theoretically unravel. We identify below a mathematical model within which to host basic computable causality. Characterising the automorphisms of this model promises to be a key task.

Here is a more recent questioning of progress towards a more comprehensive model of physics, from Peter Woit, author of the book *Not Even Wrong – The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics* (Jonathan Cape, 2006):

By 1973, physicists had in place what was to become a fantastically successful theory of fundamental particles and their interactions, a theory that was soon to acquire the name of the standard model. Since that time, the overwhelming triumph of the standard model has been matched by a similarly overwhelming failure to find any way to make further progress on fundamental questions.

And one of Peter Woit’s concerns is those undefined constants:

One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, . . .

Alan Guth, originator of the inflationary hypothesis, would like to see the laws of physics defined:

If the creation of the universe can be described as a quantum process, we would be left with one deep mystery of existence: What is it that determined the laws of physics?
If we think we are observing the universe defining its own laws, we can but hope to have access to the defining process in the course of time.

We are talking here about hugely unconventional computation. It may be so unconventional that for us it is hardly computation at all. But its existence can be framed as a something feasibly approachable, at least in principle. Roger Penrose\(^8\) calls it ‘Strong Determinism’:

[According to Strong Determinism] ... all the complication, variety and apparent randomness that we see all about us, as well as the precise physical laws, are all exact and unambiguous consequences of one single coherent mathematical structure.

In our final section, we fill in the missing ingredient — namely, the fundamental mathematical host for all this embodied information, definability and higher order computation. Before that, a remark regarding mathematical structures: Mathematical structures commonly consist of objects connected by operations or relations. Sometimes the difference between these classes is blurred, but in an interesting structure there are objects which accumulate *information* expressive of their context in the structure. Sometimes this information can be ‘read’ by the relations on the structure, which express a formal ‘causality’, whereby the distribution of information itself has a structure. This appears to be a feature of our own universe.

5 MODELLING BASIC CAUSALITY

Another quotation, this time from Lee Smolin’s 2006 book on *The Trouble with Physics*, p.241:

... causality itself is fundamental ...

The ‘early champions’ of the role of causality mentioned by Smolin – Roger Penrose, Rafael Sorkin, Fay Dowker, Fotini Markopoulou – make a doughty bunch, formidable protagonists in contemporary turf wars around quantum gravity, causal sets and a hydra-headed superstring theory. The aim, as outlined by Smolin, is a more comprehensively immanent universe\(^9\):

It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: Causal relations can determine the spacetime geometry ...Its easy to


talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. . . . We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that causality itself is fundamental – and is thus meaningful even at a level where the notion of space has disappeared.

Note that we are talking about very specific observed and computationally well-served *causality* here, which largely frees us from the strictures of John Earman\(^\text{10}\) regarding the more wide-ranging use of the term:

\[
\ldots \text{the most venerable of all the philosophical definitions [of determinism] holds that the world is deterministic just in case every event has a cause. The most immediate objection to this approach is that it seeks to explain a vague concept - determinism - in terms of a truly obscure one - causation.}
\]

In fact, a primary objective of this modelling of basic computable causality is the clarity that the mathematics of the model brings to less easily described causality, and to issues regarding over causation, downward causation and non-locality.

The question is, what kind of causality fails to engage with the informational content of the reality it structures? The relevance of the question derives from the fact that it is causal structure from which information derives and whereby it is stored. Computation is about information, and potentially equipped to model the way in which the basic causality of our universe respects and transports information. Once again it was Turing who gave us a precise formulation corresponding to the fundamentality of the intuition.

Alan Turing’s 1939 paper is a neglected masterpiece — less cited than the more famous trio of papers that gave us the universal Turing machine, the Turing test for intelligence, and the mathematics of morphogenesis — but crackling with ideas and perceptive intuitions. The *oracle Turing machine* as it came to be called appears on just one page of this densely argued article. Essentially, it equips the computer – in the form of a Turing machine – to roam the scientific universe of real numbers, accepting type 1 inputs, and outputting, if we are lucky, type 1 outputs. This sometimes described as *relative* computation.

An oracle Turing machine exactly expresses the character of basic causality in the world, progressively sampling information and transferring it comprehensively across time and space.

The mathematician or computer scientist — and maybe Turing himself at the time of its invention — regards the oracle machine as a model of how we might compute using data given to us from an unknown source. This viewpoint, together with observation of the apparent actuality of incomputability in the natural universe, provides the basis for Jack Copeland’s notion of hyper computation (beyond the Turing barrier).

But the physicist is presented with a model — the Turing universe — within which the computable content of Newtonian dynamics comfortably fits; at a basic level of course. As Poincaré speculated, and researchers from Kreisel in 1970, to Beggs, Costa and Tucker today observed, more broadly interactive contexts based on Newton’s laws can generate infinitary mathematics with attendant incomputabilities.

Mathematically, the type-2 computable functions Φ over the reals are termed Turing – or partial computable (p.c.) – functionals. Turing, despite his longterm interest in interactive computation (mainly between humans and machines), seems to have never mentioned his oracle machines again. It was left to another highly creative but under-appreciated mathematician, Emil Post, 15 years older than Turing, to set in motion the mathematical development of Turing’s model. In 1944 Post defined the degrees of unsolvability – later called the Turing degrees – as a classification of reals in terms of their relative computability.

Strangely, the subsequent investigation of the mathematical character of the Turing degree structure was a process entirely detached from reality. There was no sense at all of relevance to the real world. The fact that the Turing universe underpinned a wide range of dynamical contexts in which the ‘design’ was understood meant nothing. The possibility that all sorts of higher structure might be better understood via an analysis of definability or invariance in the basic underlying model was never entertained. I was there through a golden age of technical development. Turing was gone, taking with him his broadly questioning brilliance, leaving behind a universally adopted computational paradigm. We recursion theorists were busy doing our sums while the natural world around us computed in mysterious and wondrous ways. There was no such thing as unconventional computation.

There were mathematical events beneath which one can retrospectively detect a sort of subliminal prescience. In 1965 Hartley Rogers gave a fasci-
nating talk (judging by the 1967 paper\textsuperscript{11} that came out of it) at the Tenth Logic Colloquium in Leicester, England. What was remarkable was the focus\textsuperscript{12} on the large scale structure of the Turing universe, via the notions of invariance and definability that we have identified as relevant to the emergence of form in wide range of different environments. There was in evidence a ‘Hartley Rogers Agenda’, built around a number of deep and difficult questions about the global character of the Turing degrees. Over the years, there has been a growing intuition that Rogers’ questions are key to pinning down how higher order relations on the real world can appear to be computed. Much of the progress with these questions rests on the richness of Turing structure discovered so far. Mathematically structural pathology is a disappointment. Out in the real world pathology is super-abundant, both generator of and avatar of a richness of real world definability. In the Turing universe, the pathology takes on a parallel role, becoming the raw material for a multitude of definable relations, counterparts to visibly ‘computable’ structure out in the real world.

\section{UNDEFINABLE RELATIONS}

We might have developed the view of unconventional computation as higher type computation in various guises – emergence, definability etc – in other contexts. Another fruitful workspace would have been that of artificial intelligence/neuroscience. Each has it’s special strengths. That of the physics is the clear way in which it displays the fragility of definability, and the consequences of its failure.

Failures of definability are not necessarily negative in their impact. Many friendly features of our universe depend on them. In physics, there is a wide range of special symmetries underpinning aspects of our observed world. A symmetry is of course an instance of an automorphism at work, maybe small-scale or very selective in its scope. More important broad impact symmetries include the relationship of the SU(3) symmetry group to the quark model underlying hadrons, for which Murray Gell-Mann got a nobel prize in 1969.


One of the interests of such particular examples is that they point to the possibility of a rich automorphism structure underlying the basic causal structure, and hence to the identification on new relations defined/computed within the physics with potentially far-reaching explanatory power.

Back in the underlying Turing model, there is some disagreement about the potential character of the automorphism group of both the local and the global structures. An interest in the so-called ‘Bi-interpretability Conjecture’ originating with Leo Harrington goes back around 30 years, during which time various people have managed to prove partial versions of the conjecture, with interesting consequences for the automorphism groups. Essentially, what the conjecture says is that there is a close enough correspondence between the structure of the Turing degrees and that of second order arithmetic for the two structures to share a number of characteristics, particularly related to automorphisms and definability. A full verification of bi-interpretability would impose rigidity on the Turing universe, and invalidate it as a model for the real universe, which appears to be far from rigid. There is no consensus of informed guesses concerning rigidity.

Failure of rigidity would have potentially dramatic consequences for the longstanding search for a ‘realistic’ interpretation of quantum non locality and the collapse of the wave function in conjunction with a measurement. What we commonly have is the deterministic continuous evolution of the wave equation describing a physical system via Schrödinger’s equation, involving the superposition of basis states. We may then have a probabilistic non-local discontinuous change due to a measurement – and observe a jump to a single basis state. There are various interpretations of this. The simplest is that what we are encountering is a level of failure of definability at the ontological level of the quantum world — there is just not enough connectivity and information down there to uniquely identify basis states. While the intervention entailed by the measurement changes the situation. If it is a higher type relationship of definability gets unconventionally computed, it is allowed to operate non-locally, without any of the usual problems for the physics.

There are wider ramifications to such a ‘realistic’ and immanent deciding of physical transitions. The Many Worlds interpretation and its Multiverse derivatives begin to look pleasingly redundant. It is not good that this becomes yet another misfortune to impact on Hugh Everett III and his family – nowadays represented by his son Mark, the talented lead singer of the EELS. No doubt, even with such a powerfully persuasive replacement for
Many Worlds and the various Multiverses, via unconventional computation, it will be hard to divert David Deutsch from his view\textsuperscript{13} that:

\ldots understanding the multiverse is a precondition for understanding reality as best we can. Nor is this said in a spirit of grim determination to seek the truth no matter how unpalatable it may be \ldots It is, on the contrary, because the resulting world-view is so much more integrated, and makes more sense in so many ways, than any previous world-view, and certainly more than the cynical pragmatism which too often nowadays serves as surrogate for a world-view amongst scientists.

But there are many others, like presumably George Ellis,\textsuperscript{14} would breath a sigh of relief:

The issue of what is to be regarded as an ensemble of ‘all possible’ universes is unclear, it can be manipulated to produce any result you want \ldots The argument that this infinite ensemble actually exists can be claimed to have a certain explanatory economy (Tegmark 1993), although others would claim that Occam’s razor has been completely abandoned in favour of a profligate excess of existential multiplicity, extravagantly hypothesized in order to explain the one universe that we do know exists.

There are many other ways in which the admission of an extended computational repertoire can bolster the integrity of the observed ‘one universe that we do know exists’. Physics is just one area can benefit from the mathematics of definability, invariance, emergence and higher type computation. Alan Turing would have been fascinated.