How Computationally Relevant is Definability?
- notes from work in progress
Overview:

1. Relating observation, computation, and theory

2. The rise of emergence, quantum ambiguity, chaos, fractals, epistemological relativism, string theory, recursion theory ...

3. Descriptions versus computation

4. Towards a basic enough model of causal structure

5. Explanatory power
Newton onwards - overarching aim of science became the extraction of the algorithmic content of the world

Einstein [p.54, 'Out of My Later Years', 1950]: “When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them.”

With the constraint of Popperian falsification
Laplace’s Predictive ‘Demon’

“Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situations of the beings who compose it - an intelligence sufficiently vast to submit these data to analysis - it would embrace in the same formula the movements of the greatest bodies and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.”

from P. S. de Laplace [1819], "Essai philosophique sur les probabilités"
“For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either. ... The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem.

In contrast to the foolish Ignorabimus, our credo avers: We must know, We shall know. “

- David Hilbert’s opening address to the Society of German Scientists and Physicians, Königsberg, September 1930
A mathematical model at last

- 1936 - Turing's machines appear
- Provide a model of algorithmic natural processes within structures which are countably presented

![Diagram of Turing machine tape and reading head]

- reading head which is in internal state $q$ and obeys Turing program $P$
- tape, infinitely extendable in each direction
A mathematical model at last

- 1936 - Turing’s machines appear

- Provide a model of algorithmic natural processes within structures which are countably presented

- But - techniques for presenting machines give the Universal Turing machine - and incomputable objects
New algorithmic content ...

- Incomputable computably enumerable sets
- Approximations to $\Delta_2$ and $\Sigma_2$ sets

"... if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."

A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted by Andrew Hodges in "Alan Turing - the enigma", p.361
Natural phenomena as discipline problem

- Reduction of “natural” examples to Turing model – e.g. quantum computation

- Martin Davis versus the hypercomputationalists (Jack Copeland et al) –

“The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question ‘What is a computation?’, and, by itself, makes the existence of any more general form of computation extremely doubtful.”

Martin Davis [2004], The myth of hypercomputation. In Alan Turing: Life and legacy of a great thinker (C. Teuscher, ed.), Springer-Verlag
Descriptive versus computational content

- Mathematical route to incomputability is descriptive, not through direct analysis of higher computational content.
- Hypercomputationists tend to work via computational models in which the role of definability is not explicit.
- Only relativistic computation using black holes escapes Martin Davis’ reductionism.
- Need is to translation of descriptive content in the material world into explicitly computability-theoretic structures.
QUESTIONS . . .

☐ Can deconstruction of natural processes provide us with genuinely new ways of computing?

☐ To what extent can descriptive formulations of natural processes be captured computationally?

☐ Is there a universal model underlying such new computational paradigms?

☐ If not, is there a new kind of model with explanatory potential?
Signals from the real world -

- Persistence of problems of predictability - at quantum level, relation between emergence and chaos, relativistic phenomena (see Istvan Németi and Hajnal Andreka, 2005)

- Renewed interest in analog and hybrid computing machines leading to: “... the classical Turing paradigm may no longer be fully appropriate to capture all features of present-day computing.”

“Von Neumann’s axioms distinguished the U (unitary evolution) and R (reduction) rules of quantum mechanics. Now, quantum computing so far (in the work of Feynman, Deutsch, Shor, etc) is based on the U process and so computable. It has not made serious use of the R process: the unpredictable element that comes in with reduction, measurement, or collapse of the wave function.”

Andrew Hodges

in “What would Alan Turing have done after 1954?”, from Teuscher,
“Alan Turing: Life and legacy of a great thinker”
Co-operative phenomena

- 1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in "an analog computation of a non-recursive function"

- Painlevé Problem (1897): Do non-collision singularities exist for the N-body problem for any \( N \geq 4 \)?

- "Yes" - Jeff Xia, 1988, Saari and Xia, Off to Infinity in Finite Time, Not. Amer. Math. Soc. 42, 1995
Chaos and its Analogues

- Growth of **Chaos theory**, generation of informational complexity via very simple rules, accompanied by the emergence of new regularities - e.g. Robert Shaw [1984]

- **Link** between structures in nature, and mathematical objects, such as the Mandelbrot and Julia sets

- Penrose, Smale - **computability** of Mandelbrot, Julia sets?
Now we witnessed ... a certain extraordinarily complicated looking set, namely the Mandelbrot set. Although the rules which provide its definition are surprisingly simple, the set itself exhibits an endless variety of highly elaborate structures.

Roger Penrose
in “The Emperor’s New mind”, Oxford Univ. Press, 1994

Recent results - Braverman [1999], Hertling [2005], Rettinger [2005], Rettinger and Weihrauch [2003]
Computation outside the standard model?

“One example of a problem that is not algorithmic is the following instruction from a recipe [quote Knuth, 1968]:

‘toss lightly until the mixture is crumbly.’

This problem is not algorithmic because it is impossible for a computer to know how long to mix: this may depend on conditions such as humidity that cannot be predicted with certainty ahead of time.

In the function-based mathematical worldview, all inputs must be specified at the start of the computation, preventing the kind of feedback that would be necessary to determine when it’s time to stop mixing.”

The elusive Global to Local connection

- **Extreme example** - The actuality of quantum non-locality, and lack of understanding of its connection with familiar physical laws (EPR, Bell’s Inequality)

- **Closer to home** - Economic activity (e.g.) and its global constraints

  - Weather prediction, evolution, consciousness, of classical reality from quantum ambiguity, the origins of life, or of large structures in the Universe . . .
Descriptions and Emergent Structure

- **Intuition** - entities exist because of, and according to, mathematical laws. In the words of Leibniz [1714] -

  ‘The Monadology’, sections 31, 32: “... there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases.”
Definability and Invariance

- So natural phenomena not only generate descriptions, but arise and derive form from them . . .

- ... so connecting with a useful abstraction - that of mathematical definability - or, more generally, invariance (under the automorphisms of the appropriate structure) . . .

- So providing a potentially non-algorithmic determinant of events
I believe the following aspects of evolution to be true, without knowing how to turn them into (respectable) research topics.

Important steps in evolution are robust. Multicellularity evolved at least ten times. There are several independent origins of eusociality. There were a number of lineages leading from primates to humans. If our ancestors had not evolved language, somebody else would have.

Cooperation and language define humanity. Every special trait of humans is a derivative of language.
“According to Bacon, scientists should travel over the earth collecting facts, until the accumulated facts reveal how Nature works. The scientists will then induce from the facts the laws that Nature obeys. According to Descartes, scientists should stay at home and deduce the laws of Nature by pure thought. ... Faraday and Darwin and Rutherford were Baconians: Pascal and Laplace and Poincaré were Cartesian. Science was greatly enriched by the cross-fertilization of the two contrasting ... cultures.”

from Freeman Dyson’s introduction to George Odifreddi’s “The Mathematical Century: The 30 Greatest Problems of the Last 100 Years”
The Human Mind As Case Study

- **Familiar** - Baconian experience easily got through solving everyday problems, and observing others

- **Mechanics** of brain well-documented

- **Does not feel, or appear to compute, like a Turing machine** - role of creativity, consciousness, intuition

- **Relevant** - importance of *copying* how humans think for AI etc... and intuition that a physical brain only reflects processes in the wider universe
"Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience sake, I verified the result at my leisure."

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press
Intelligent machines as emergent phenomena

Need to bridge the gap between higher mental functionality and ... what algorithmic context?

Difficult - Rodney Brooks [Nature, 2001]: “neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant.”

But paradigm-stretching features in evidence in many different contexts . . .
Nature in the driving seat

- Getting intelligent machines themselves via emergence...

“I used to think we'd do it by engineering. Now I believe we'll evolve them. We're likely to make thinking machines before we understand how the mind works, which is kind of backwards.”

- Daniel Hillis, Chief Technology Officer of Applied Minds, Inc. (and ex-Vice President, Research and Development at Walt Disney Imagineering), April 2001
There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.

*Paul Smolensky* [1988] (recipient 2005 David E. Rumelhart Prize),

*On the proper treatment of connectionism,* in *Behavioral and Brain Sciences, 11,* pp. 1-74
Connectionist Models of Computation?

- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets.

- But for Steven Pinker "... neural networks alone cannot do the job".

- And focusing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion"...
We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

Steven Pinker,


Making a similar point - Damasio has a nice description of the hierarchical development of a particular instance of consciousness within the brain, interacting with some external object . . .
“... both organism and object are mapped as neural patterns, in first-order maps; all of these neural patterns can become images. ... The sensorimotor maps pertaining to the object cause changes in the maps pertaining to the organism. ... [These] changes ... can be re-represented in yet other maps (second-order maps) which thus represent the relationship of object and organism. ... The neural patterns transiently formed in second-order maps can become mental images, no less so than the neural patterns in first-order maps.”

Antonio Damasio,
The Feeling Of What Happens, Harcourt, Orlando FL, 1999

- Picture is - re-representation of neural patterns formed across some region of the brain, in such a way that they can have a computational relevance in forming new patterns

- Key conception - computational loops incorporating, in a controlled way, these ‘second-order’ aspects of the computation itself
Towards a basic computational model

- **Key ingredients** - imaging, parallelism, interconnectivity, and a counterpart to the second-order recursions pointed to above

- **Connectionist models** - strong on parallelism, interconnectivity, imaging - but not recursions
Turing on Description versus Computation

- Turing, 1939 - The computational content of descriptions can be captured hierarchically - but in unpredictable ways

- No consistent theory captures arithmetic (Gödel) - but we can hierarchically transcend this barrier

- But then - identifying the route to a new theorem involves using an incomputable oracle

- Despite inductive structure, reductionism breaks down
Mathematical reasoning may be regarded ... as the exercise of a combination of ... intuition and ingenuity. ... In pre-Gödel times it was thought by some that all the intuitive judgements of mathematics could be replaced by a finite number of ... rules. The necessity for intuition would then be entirely eliminated.

In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but ingenuity, and this in spite of the fact that our aim has been in much the same direction.

Alan Turing [1939],


☐ An explanation of why written proofs do not tell us how the proof was discovered ...
The Turing model extended

- 1939 - Turing’s oracle Turing machines appear

- Provides a model of algorithmic content of structures which are based on the reals

A familiar picture:

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0 0 1 1 0 0 0 . . .. . .
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reading head which is in internal state $q$ and obeys Turing program $P$

tape, infinitely extendable in each direction
The Turing model extended

☐ 1939 - Turing’s oracle Turing machines appear

☐ Provides a model of algorithmic content of structures, based on p.c. functionals over the reals

☐ 1944 - Post defines the degrees of unsolvability as a classification of reals in terms of their relative computability

☐ ... giving a landscape one can zoom in or out of
Can describe global relations in terms of local structure ...

... so capturing the emergence of large-scale structure

Mathematically - formalise as definability over structure based on Turing functionals

More generally - as Invariance under automorphisms
Hartley Rogers’ programme ...

Fundamental problem: Characterise the Turing invariant relations
Hartley Rogers’ programme ...

**Fundamental problem:** Characterise the Turing invariant relations

**Intuition:** These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure
Fundamental problem: Characterise the Turing invariant relations

Intuition: These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure.

Notice: The richness of Turing structure discovered so far becomes the raw material for a multitude of non-trivially definable relations.
A foundational crisis in physics

By 1973, physicists had in place what was to become a fantastically successful theory of fundamental particles and their interactions, a theory that was soon to acquire the name of the ‘standard model’. Since that time, the overwhelming triumph of the standard model has been matched by a similarly overwhelming failure to find any way to make further progress on fundamental questions.

Introduction to Peter Woit: “Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics”, Jonathan Cape, 2006
... I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature ... 

nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) ...
Peter Woit: “One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ...”
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String theory as a unifying explanatory theory - “the only game in town” … ?

The longstanding crisis of string theory is its complete failure to explain or predict any large distance physics. … String theory is incapable of determining the dimension, geometry, particle spectrum and coupling constants of macroscopic spacetime. … The reliability of string theory cannot be evaluated, much less established. String theory has no credibility as a candidate theory of physics.

Lee Smolin’s 5 Great Problems:

1. Combine general relativity and quantum theory into a single theory that can claim to be the complete theory of nature.

2. Resolve the problems in the foundations of quantum mechanics.

3. The unification of particles and forces problem: Determine whether or not the various particles and forces can be unified in a theory that explains them all as manifestations of a single, fundamental entity.

4. Explain how the values of the free constants in the standard model of physics are chosen in nature.

5. Explain dark matter and dark energy. Or, if they don’t exist, determine how and why gravity is modified on large scales.

The state of physics today is like it was when we were mystified by radioactivity ... They were missing something absolutely fundamental. We are missing perhaps something as profound as they were back then.
“Causality is fundamental”

- Early champions of the role of causality - Roger Penrose, Rafael Sorkin, Fay Dowker, and Fotini Markopoulou

It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: Causal relations can determine the spacetime geometry ...

It’s easy to talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. ... We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that causality itself is fundamental - and is thus meaningful even at a level where the notion of space has disappeared.

Lee Smolin, The Trouble With Physics, p.241
A deconstructed informational Universe

Described in terms of reals ... With natural laws based on algorithmic relations between reals

- Emergence described in terms of definability/invariance
- ... with failures of definable information content modelling quantum ambiguity
- ... which gives rise to new levels of algorithmic structure
- ... and a fragmented scientific enterprise
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### Definability in physical and mathematical contexts...

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Thank you!