RECURSIVE FUNCTION THEORY: NEWSLETTER

No. 1. April, 1972.

The Newsletter is an informal means of circulating information amongst recursion theorists and others interested in recursive function theory. Results and other announcements should be kept fairly brief and sent to:

Barry Cooper/Dick Epstein,
Group in Logic and the Methodology of Science,
Department of Mathematics,
University of California,
Berkeley,
Cal. 94720,
U.S.A.

(a) Complexity of recursive functions and subrecursive hierarchies.

1. **Lemma 0:** (nothing new) For one tape Turing Machine measure of complexity, and for all partial recursive functions f ,

$$\phi_e = f \Rightarrow \exists K_e \ni \Phi_e(n) \geqslant K_e \log f(n) \log \log f(n) \ , \ \forall n \ .$$

Lemma 1: There are arbitrarily large total recursive functions f , such that

$$\exists \phi_e = f \text{ and } \Phi_e(n) \leqslant M \log f(n) \log \log f(n) \ , \ \forall n$$

which shows that the bound in Lemma 0 is best possible over the entire class of recursive functions (otherwise, it might be the case that for sufficiently large functions, the bound in lemma 0 could be strengthened.)

Theorem: Let C_t be the complexity class of one tape Turing Machines for time function $t(n)$. Then there exist arbitrarily large total recursive $t(n)$ such that

$$C_t \neq \cup\{C_{t'} : C_{t'} \subsetneqq C_t\}$$

This theorem is a kind of companion to the Union Theorem.

[Mark Finkelstein, Department of Mathematics,
University of California,
Irvine, California 92664, U.S.A.]

2. There is an absolute upper limit to the number of applications of the uniqueness rules required in any proof in the equation calculus.

[R.L. Goodstein, The University, Leicester, England.]

3. We let $\phi_i^{(A)}$ represent the i^{th} partial A-recursive function, and consider the complexity of such functions for various sets A . The general approach we use is similar to that of Blum (1) for dealing with the complexity of partial recursive functions.

For convenience, we speak here in terms of the space measure:

$$S_i^{(A)}(x) = \begin{cases} \text{the number of worktape squares used by} \\ \text{oracle Turing Machine } i \text{ with oracle } A \\ \text{on input } x \text{, if } \phi_i^{(A)}(x) \text{ converges} \\ \qquad\qquad\qquad\qquad \text{undefined otherwise} \end{cases}$$

(1) It is clear that some oracle sets make the computation of some functions require less space than it would

without the oracle. We say such oracle sets "help" the computation of the function.

We consider the following (informal) question: For every recursive set A, do there exist arbitrarily complex recursive sets B not helping the computation of A's characteristic function? Some results of this general type have been obtained by Machtey, for primitive recursive classes (2).

The strongest result we have thus far follows after a few definitions:

<u>Definitions</u>:

"a.e." means for all except (possibly) finitely many values

"$\mathrm{Comp}^{(B)}\ A > t$ a.e." means $(\forall i)[\phi_i^{(B)} = C_A \Rightarrow S_i^{(B)} > t \text{ a.e.}]$

"$\mathrm{Comp}\ A > t$ a.e." means $\mathrm{Comp}^{(\emptyset)}\ A > t$ a.e.

"$\mathrm{Comp}\ A \leqslant t$ a.e." means $(\exists i)[\phi_i^{(\emptyset)} = C_A \wedge S_i^{(\emptyset)} \leqslant t \text{ a.e.}]$

We say a total function t is "constructable" if $(\exists i)[\phi_i^{(\emptyset)} = t \wedge S_i^{(\emptyset)} \leqslant t]$.

We say a property holds "for arbitrarily complex recursive sets B" if for every total recursive function r, there exists a recursive set B having the required property and such that $\mathrm{Comp}\ B > r$ a.e.

<u>Theorem</u>: Assume we have a recursive set A and a constructable function t. Assume

$$\text{Comp } A > 2^t \text{ a.e.}$$

and

$$\text{Comp } A \leqslant \lambda x[t(x+1)] \text{ a.e.}$$

Then there exist arbitrarily complex recursive sets B such that

$$\text{Comp}^{(B)} A > t \text{ a.e.}$$

It is still unknown whether all the bounds on A's complexity are needed and also unknown whether B may be obtained independently of t.

(2) We obtain, as an immediate corollary to a relativization of the Union Theorem (3) the following result:

Theorem: To any countable collection $\mathcal{B}$ of sets, there corresponds a single function t such that

$$(\forall B \in \mathcal{B})(\forall A)[A \underset{++}{\leqslant} B \Leftrightarrow (\exists i)(C_A = \phi_i^{(B)} \wedge S_i^{(B)} \leqslant t \text{ a.e.})]$$

References:

(1) Blum, M., A machine-independent theory of complexity of recursive functions, *JACM*, *vol.* *14* (1967), 322-336.

(2) Machtey, M., Augmented loop languages and classes of computable functions, Indiana University Computer Science Department, Bloomington, Ind. Dec. 1971.

(3) E.M. McCreight and A.R. Meyer, Classes of computable functions defined by bounds on computation: preliminary report, Conference Record of the ACM Symposium on Theory of Computing, Association for Computing Machinery, New York, 1969, pp. 79-88.

[Mrs. Nancy Lynch, Automata Theory Group,
Project Mac, M.I.T.,
Cambridge, Mass. 02139, U.S.A.]

4. $\{D_n\}$ is a canonical effective enumeration of the finite subsets of ω .

Theorem A. Let P be a recursive partial ordering on ω . Then the following three conditions are equivalent:

(i) For every recursive f $\exists$ recursive f' $\ni$

$$\text{(a)} \quad f'(x) > f(x) \; , \quad \forall x \in \omega$$

$$\text{(b)} \quad x <_p y \Rightarrow f'(x) < f'(y)$$

(ii) $\exists$ a recursive function g $\ni$

$$D_{g(x)} = \{y \mid y <_p x\} \; , \quad \forall x \in \omega$$

(iii) $\exists$ a recursive permutation, h , of ω $\ni$

$$x <_p y \Rightarrow h(x) < h(y)$$

For the following result $\langle\cdot\rangle$ is a fixed effective one-one map of $\bigcup_{K=1}^{\infty} \omega^K$ onto ω , while $\langle\ \rangle_K$ is a fixed effective one-one map of ω^K onto ω.

Result: For any computational complexity measure Φ $\exists$ a rec. fn. B of 3 variables such that if

$$F(x) = [\mu y][G(x, y) = 1] \text{ where } G \text{ is total rec.}$$

$$\Phi_g(\langle x, y\rangle_2) = G(x, y) \text{ for some recursive function } G$$

then $\exists$ f such that $\phi_f = F$ and

$$\phi_f(x) \leqslant B(\langle \Phi_g(\langle x,0\rangle_2),\ldots,\Phi_g(\langle x,F(x)\rangle_2)\rangle,\ F(x),\ x)$$

for all but finitely many x such that F(x) is defined. Moreover, B can be chosen to be increasing in all its variables. Analogous results hold for composition and primitive recursion.

Theorem B: Let f be a partial recursive function. The following two conditions are equivalent:

(i) gr f is recursive

(ii) $\exists$ a rec. fn. e of 2-variables such that if ϕ_j is recursive, dominates an unbounded non-decreasing recursive function and $\phi_i = f$, then $\phi_{e(i,j)}$ is total and whenever $\phi_w = f$ we have

$$\Phi_i(x) \leqslant \phi_{e(i,j)}(\Phi_w(x),x), \quad \text{for } x > \phi_j(w).$$

[Paul Morris, Dept. of Mathematics,
University of California,
Irvine, Cal. 92664, U.S.A]

(b) Constructive ordinals.

5. If $a \in 0$ (Kleene's set of notations for constructive ordinals) let $0(a) = \{b: b <_0 a\}$. Then there exists $a \in 0$ such that $|a| = \omega^2$ and $0(a)$ is nonrecursive. By contrast there exists a π_1^1 path P through 0 such that $0(a)$ is uniformly recursive for $a \in P$.

[C. Jockusch, Department of Mathematics,
University of Illinois,
Urbana, Ill. 61801, U.S.A.]

(c) Recursively enumerable sets.

6. For any set α, let θA^α denote the index set of the class of all r. e. subsets of α. Such sets appear to have interesting properties in the partial ordering of one-one degrees of index sets. To what extend do α and θA^α determine each other?

Theorem. If $\underset{\sim}{b}$ is a non-recursive r. e. degree, the Turing degrees of sets θA^α for α r. e., $\alpha \in \underset{\sim}{b}$ are exactly the degrees $\underset{\sim}{c} \geqslant \underset{\sim}{0}'$ such that $\underset{\sim}{c}$ is r. e. in $\underset{\sim}{b}$. Thus at least in the case where α is r. e., the Turing degrees of α and θA^α are essentially independent.

[Louise Hay, University of Illinois at Chicago Circle,
College of Liberal Arts and Sciences,
Dept. of Mathematics (Box 4348),
Chicago, Ill. 60680, U.S.A.

7. No maximal (or even dense simple) set is strongly effectively simple (s.e.s.). (A simple set A is s.e.s. if there is a recursive function g such that $W_e \subseteq \overline{A} \Rightarrow \max(W_e) \leqslant g(e))$. (By contrast it is routine to show that a maximal set can be effectively simple.)

From this it follows that Post's simple set is not contained in any maximal set. In the other direction, P.F. Cohen (same address) has shown that any coinfinite r.e. set which is not dense simple is contained in some s.e.s. set. From this it follows that there exist r-maximal s.e.s. sets.

[C. Jockusch, Dept. of Mathematics,
University of Illinois,
Urbana, Ill. 61801, U.S.A.]

(d) <u>Degrees of unsolvability</u>.

8. Let $\underset{\sim}{a}$ be a r.e. degree for which $\underset{\sim}{a}' = \underset{\sim}{0}''$. Then there is a minimal pair of r.e. degrees less than $\underset{\sim}{a}$. (It is known that there is no uniform procedure for obtaining a minimal pair below <u>any</u> given r.e. $\underset{\sim}{a} > \underset{\sim}{0}$).

[S.B. Cooper, Dept. of Mathematics,
University of California,
Berkeley, California, U.S.A.

9. There is a degree $\underset{\sim}{a}$ such that every $\underset{\sim}{b} \geqslant \underset{\sim}{a}$ is a minimal cover. (The degree $\underset{\sim}{b}$ is a <u>minimal cover</u> if

$(\exists \underset{\sim}{c} < \underset{\sim}{b})(\forall \underset{\sim}{d}) \neg [\underset{\sim}{c} < \underset{\sim}{d} < \underset{\sim}{b}]$.) This result is proved in ZF as a Corollary to Paris' recent result that $\underset{\sim}{\Sigma}{}^0_4$ sets are determinate. It follows from this result that the degrees of arithmetical sets (as a partially ordered structure without the jump operation) do not form an elementary substructure of all degrees. (Cf. "Minimal covers and arithmetical sets," A.M.S. Proceedings, *25* (1970), 856-859.)

[C. Jockusch, Dept. of Mathematics,
University of Illinois,
Urbana, Ill. 61801, U.S.A.]

10. There is a minimal partial degree (*not* enumeration degree) below $\underset{\sim}{0}'$.

[L.P. Sasso, Jr., Dept. of Mathematics,
University of California,
Irvine, Cal. 92664, U.S.A.]

11.

$(\tau, \underset{\sim}{d})$-abundance

Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{S}$ the set of strings of zeros and ones. We introduce a notion of $(\tau, \underset{\sim}{d})$-*abundance* for subsets of $2^{\mathbb{N}}$, where τ is a *tree-system*, i.e., a set of subsets of $\mathbb{S}$ satisfying some simple conditions. Every $(\tau, \underset{\sim}{d})$-abundant set is abundant, (i.e., comeager, residual, of second category) relative to the topology induced by τ . Neighborhoods are of the form

$\mathfrak{n}(T) = \{B\colon B \text{ lies on the tree } T\}$. T is perfect if $\mathfrak{n}(T)$ is a perfect subset of $2^{\mathbb{N}}$ relative to the usual topology. For any system τ, let τ^P be the subsystem of perfect trees; τ is a perfect system if $\tau = \tau^P$. τ^* is the subsystem of simple trees, where a simple tree is of the form $\{\sigma\colon \beta \leqslant \sigma \wedge \sigma \in U(\tau)\}$ and $U(\tau)$ is the maximum element of the system τ (usually $\$$) under $\subseteq$. The early work of Kleene and Post was confined to simple trees, as are the easier priority theorems concerning $\sum_1$ sets.

Theorem 1 (First Genericity Theorem). If τ is a perfect $\underset{\sim}{d}$-system (i.e., all its elements are of degree $\leqslant \underset{\sim}{d}$) then every $(\tau, \underset{\sim}{d})$-abundant set is dense in the τ-topology, has the cardinality of the continuum and contains a function of degree $\leqslant \underset{\sim}{d}$.

Theorem 2 (Second Genericity Theorem). If $\underset{\sim}{x} \geqslant \underset{\sim}{d}$ and τ is a perfect $\underset{\sim}{d}$-system, where $\underset{\sim}{d} \geqslant \underset{\sim}{0}^{(1)}$, then every $(\tau, \underset{\sim}{d})$-abundant set contains an element of degree $\underset{\sim}{b}$ such that $\underset{\sim}{b}^{(1)} \leqslant \underset{\sim}{x}$ and $\underset{\sim}{b} \cup \underset{\sim}{d} = \underset{\sim}{x}$; in particular, if $\underset{\sim}{d} = \underset{\sim}{0}^{(1)}$ then $\underset{\sim}{b}^{(1)} = \underset{\sim}{x}$.

This generalizes Friedberg's theorem on the jump operator.

There follow a number of results about general $(\tau, \underset{\sim}{d})$-abundance, after which we enumerate some results in which particular tree-systems τ and degrees $\underset{\sim}{d}$ have to be chosen.

<u>Theorem 3</u> (Uniformity Theorem). Let τ be a perfect $\underset{\sim}{d}$-system. If $A \subseteq 2^N$ is uniformly of degree $\leqslant \underset{\sim}{d}$ then A is $(\tau, \underset{\sim}{d})$-meager.

<u>Theorem 4</u>. Let τ be a perfect $\underset{\sim}{c}^{(1)}$-system. Then $\mathbb{D}(\leqslant \underset{\sim}{c})$ is $(\tau, \underset{\sim}{c}^{(1)})$-meager. Also, if $(\underset{\sim}{a}_n)$ is uniformly of degree $\leqslant \underset{\sim}{c}$ then $\cup\, \mathbb{D}(\underset{\sim}{a}_n)$ is $(\tau, \underset{\sim}{c}^{(1)})$-meager.

This result may be significantly improved in terms of the notion of priormeager set (see second abstract). It implies in particular that the set of recursive functions is $(\tau, \underset{\sim}{0}^{(1)})$-meager for any perfect $\underset{\sim}{0}^{(1)}$-system τ. This represents the usual diagonalization technique.

<u>Theorem 5</u>. Let τ be a perfect $\underset{\sim}{c}$-system. If $(\underset{\sim}{a}_n)$ is uniformly of degree $\leqslant \underset{\sim}{c}^{(1)}$ then $\bigcup_{\underset{\sim}{a}_n \not\leqslant \underset{\sim}{c}} \mathbb{D}(\geqslant \underset{\sim}{a}_n)$ is $(\tau, \underset{\sim}{c}^{(1)})$-meager. In particular, if $\underset{\sim}{c} < \underset{\sim}{a} < \underset{\sim}{c}^{(1)}$ then $\mathbb{D}(\geqslant \underset{\sim}{a})$ is $(\tau, \underset{\sim}{c}^{(1)})$-meager.

This implies that the set

$$\{B: \text{there is a nonrecursive } \textstyle\sum_1 \text{ set recursive in } B\}$$

is $(\tau, \underset{\sim}{0}^{(1)})$-meager, for any perfect recursive system τ. From this we immediately obtain an old theorem of Shoenfield: there are nonzero degrees below $\underset{\sim}{0}^{(1)}$ with no nonzero $\sum_1$ predecessors. Again, the notion of priormeager set enables us to strengthen this result.

Now, we turn to specialized results. Let $\mathbb{T}$ be the system of all trees, T_0 the system of all recursive trees; one prime interest lies in $T_0{}^P$ and $T_0^* = T^*$.

<u>Theorem 6</u>. The set

$$\{B\colon B_{\langle 0\rangle}, B_{\langle 1\rangle} \text{ form a minimal pair}\}$$

is $(T^*, \underset{\sim}{0}^{(1)})$-abundant.
(Here, $B_{\langle 0\rangle}(n) = B(2n)$ and $B_{\langle 1\rangle}(n) = B(2n+1)$ for all n.) Hence the set $\mathcal{M}$ of functions of minimal degree is <u>not</u> $(T^*, \underset{\sim}{0}^{(1)})$-abundant. On the other hand:

<u>Theorem 7</u>. $\mathcal{M}$ is $(T_0{}^P, \underset{\sim}{0}^{(2)})$-abundant.

This result can be improved using priorabundance instead of abundance to replace $\underset{\sim}{0}^{(2)}$ by $\underset{\sim}{0}^{(1)}$ (see the Second Abstract).

From these theorems we obtain existence theorems using the First Genericity theorem: a continuum of minimal degrees, a minimal degree below $\underset{\sim}{0}^{(2)}$ and a minimal pair below $\underset{\sim}{0}^{(1)}$.

A result of limited interest concerning the jumps of minimal degrees can be obtained using the Second Genericity Theorem: if $\underset{\sim}{d} \geqslant \underset{\sim}{0}^{(2)}$ then there is a minimal degree $\underset{\sim}{b}$ such that $\underset{\sim}{b}^{(1)} \leqslant \underset{\sim}{d} \leqslant \underset{\sim}{b}^{(2)}$. To replace the latter clause by $\underset{\sim}{b}^{(1)} = \underset{\sim}{d}$ requires $(\tau, \underset{\sim}{d})$-priorabundance (see the Second Abstract). To further replace $\underset{\sim}{0}^{(2)}$ by $\underset{\sim}{0}^{(1)}$ requires $(\tau, \lim(\underset{\sim}{d}))$-priorabundance (in preparation). This strongest possible theorem concerning the jumps of minimal degrees is due originally to S.B. Cooper.

Abundance may be used to show that every degree $\geqslant \underset{\sim}{0}^{(2)}$ is the join of two minimal degrees; priorabundance is required to replace $\underset{\sim}{0}^{(2)}$ by $\underset{\sim}{0}^{(1)}$ but one can achieve this simply with $(\tau, \underset{\sim}{d})$-priorabundance, without having to resort to $(\tau, \lim(\underset{\sim}{d}))$-priorabundance as Cooper implicitly did in his original proof of this result. Different tree-systems have to be used in order to formulate relativization and the various results concerning the join operation and upper bounds for sequences of degrees. Lack of space prevents us from discussing these in detail here.

[C.E.M. Yates, Dept. of Mathematics,
University of Colorado,
Boulder, Col. 80302, U.S.A.]

12.

$(\tau, \underset{\sim}{d})$-priorabundance

We introduce a notion of $(\tau, \underset{\sim}{d})$-priorabundance for subsets of $2^{\mathbb{N}}$, corresponding to problems requiring the Priority Method. For perfect tree-systems τ (see preceding abstract), this fits in neatly between $(\tau, \underset{\sim}{d})$-abundance and $(\tau, \underset{\sim}{d}^{(1)})$-abundance and in general presents a method of reducing the degree of a problem.

Theorem 1 (First Prioricity Theorem). If τ is a $\underset{\sim}{d}$-system then every $(\tau, \underset{\sim}{d})$-priorabundant set is dense over τ^{P}, has the cardinality of the continuum and contains a function of degree $\leqslant \underset{\sim}{d}$.

The proof of this result requires the simple (finite-injury) Priority Method, as does the second theorem below. None of the subsequent theorems require the Method since it is fully represented by these theorems.

Theorem 2 (Second Prioricity Theorem). If $\underset{\sim}{x} \geqslant \underset{\sim}{d}^{(1)}$ and $\underset{\sim}{d} \geqslant \underset{\sim}{0}^{(1)}$ then every $(\tau, \underset{\sim}{d})$-priorabundant set contains an element of degree $\underset{\sim}{b}$ such that $\underset{\sim}{b}^{(1)} \leqslant \underset{\sim}{x}$ and $\underset{\sim}{b} \cup \underset{\sim}{d} = \underset{\sim}{x}$; in particular, if $\underset{\sim}{d} = \underset{\sim}{0}^{(1)}$ then $\underset{\sim}{b}^{(1)} = \underset{\sim}{x}$.

This implies some of the generalisations of Friedberg's theorem on the jump operator which require the Priority Method.

There follow a number of results about general $(\tau, \underset{\sim}{d})$-priorabundance, after which we enumerate some results in which particular tree-systems and degrees have to be chosen. Of the general results, Theorem 4 is the most important since it represents an improvement over Theorem 4 of the First Abstract, this improvement requiring an essential use of the Priority Method (via Theorem 1 above).

Theorem 3. Let τ be a $\underset{\sim}{d}$-system. If $A \subseteq 2^N$ is uniformly of degree $\leqslant \underset{\sim}{d}$ then A is $(\tau, \underset{\sim}{d})$-priormeager.

Theorem 4. Let τ be a $\underset{\sim}{c}$-system and $\underset{\sim}{a} < \underset{\sim}{c}^{(1)}$. Then $\mathbb{D}(\leqslant \underset{\sim}{a})$ is $(\tau, \underset{\sim}{c}^{(1)})$-priormeager. More generally, if $(\underset{\sim}{a}_n)$ is uniformly of degree $\leqslant \underset{\sim}{c}^{(1)}$ then $\bigcup_{\underset{\sim}{a}_n < \underset{\sim}{0}^{(1)}} \mathbb{D}(\leqslant \underset{\sim}{a})$ is $(\tau, \underset{\sim}{c}^{(1)})$-priormeager.

One consequence of this result is our old theorem that there is a degree $< \underset{\sim}{0}^{(1)}$ which is incomparable with <u>all</u> the $\sum_1$ degrees between $\underset{\sim}{0}$ and $\underset{\sim}{0}^{(1)}$. In fact, using the next theorem below, this degree can also be made minimal (a result due to Sasso). $\mathcal{M}$ consists of the functions of minimal degree.

Theorem 5. $\mathfrak{M}$ is $(T_0, \underset{\sim}{0}^{(1)})$-priorabundant.

Moreover, we can also derive Shoenfield's theorem that for any $\underset{\sim}{a}$ s.t. $\underset{\sim}{0} < \underset{\sim}{a} < \underset{\sim}{0}^{(1)}$ there is a minimal degree incomparable with $\underset{\sim}{a}$ and below $\underset{\sim}{0}^{(1)}$.

Cooper has obtained a stronger theorem than Shoenfield's: $\underset{\sim}{0}^{(1)}$ is the join of two minimal degrees. Using $(\tau, \underset{\sim}{d})$-priorabundance we prove that every degree $\geqslant \underset{\sim}{0}^{(1)}$ is the join of two minimal degrees.

All other results concerning minimal degrees appear to make essential use of recursive approximation and hence a stronger notion of priorabundance which we call $(\tau, \lim(\underset{\sim}{d}))$-priorabundance. Details of this will be made available later.

One further application of the Priority Method to $\mathbf{D}(\Delta_2)$ can, however, be deduced within the framework above given a few minor modifications: this is Shoenfield's result that if $\underset{\sim}{d} \geqslant \underset{\sim}{0}^{(1)}$ and $\underset{\sim}{d}$ is in $\sum_1(\underset{\sim}{0}^{(1)})$ then $\underset{\sim}{d}$ is the jump of a degree below $\underset{\sim}{0}^{(1)}$. Sacks of course extended this result many years ago but it remains an interesting further application of $(\tau, \underset{\sim}{d})$-priorabundance.

[C.E.M. Yates, Dept. of Mathematics,
University of Colorado,
Boulder, Col. 80302, U.S.A.]

13. Proximity of k-sections

U and V are objects of finite type. $S_k U$ (the k-section of U) is the set of all objects of type k recursive in U. ${}^nE$ is the equality predicate restricted to objects of type less than n.

Theorem. If U is of type n, ${}^nE$ is recursive in U, and k is less than n, then there exists a V of type $k+1$ such that ${}^{k+1}E$ is recursive in V and $S_k U = S_k V$. The proof opens with a downward Skolem-Löwenheim construction that defines the class of forcing conditions needed to erect a certain generic class (rather than set) that serves as the "kernel" of V.

[Gerald E. Sacks, Dept. of Mathematics,
M.I.T.,
Cambridge, Mass. 02139, U.S.A.]

(f) Special communications.

14. Professor J.N. Crossley (Dept. of Mathematics, Monash University, Clayton, Victoria 3168, Australia) would be interested to learn of any new publications on RET's, in order to update the bibliography of his survey article in the Bulletin of the London Mathematical Society.

15. SYMPOSIUM ON GENERALIZED RECURSION THEORY
OSLO, JUNE 1972.

(1) The Symposium will be arranged under the sponsorship of the Norwegian Mathematical Association and the Institute of Mathematics of the University of Oslo. The Symposium will be supported by a grant from the Norwegian Research Council.

The committee in charge of the arrangement consists of: Peter Hinman and Jens Erik Fenstad.

(2) The Symposium will take place at the Institute of Mathematics, University of Oslo, in the week June 12 to June 16, 1972.

An average of three lectures pr. day are planned, so there will be ample time for discussion and "private" work sessions.

The Institute of Mathematics will provide the necessary facilities (including secretarial assistance).

(3) The Symposium will be devoted to generalizations of recursion theory and applications of these. Of particular interest are topics from axiomatic recursion theory, hierarchy theory,

theory of inductive definitions, higher order functionals and constructibility theory.

(4) The following people will give invited addresses:

USA: J. Barwise, T. Grilliot, W. Richter,
Y. Moschovakis, G. Sacks, S. Simpson.
England: P. Aczel, R. Gandy.
France: F. Ville

The Symposium will also be open to other interested persons. However, there are no possibilities of financial aid to non-invited participants.

(5) Further inquiries concerning the Symposium should be addressed to

Professor J.E. Fenstad
Institute of Mathematics
University of Oslo
Blindern, Oslo 3
Norway