NEED TO SHOW — There exists a d.c.e. degree $d$ which is not discretely splittable over $a$ avoiding $b$, some $a, b < d$.

THE REQUIREMENTS:

$$\begin{align*}
\mathcal{P}_\Theta & : \hspace{1em} B \neq \Theta^A \\
\mathcal{Q}_{\Psi, \Phi, \Theta, \Xi, W, \overline{W}} & : \hspace{1em} D = \Psi(\Phi^{A,B,D}, \Phi^{A,B,D}, A) \\
\Rightarrow & \hspace{1em} B = \Gamma^{\Phi^{A,B,D}, A} \lor B = \Lambda \Phi^{A,B,D}, A \\
\lor & \hspace{1em} \Omega(\Phi^{A,B,D}, A) \neq \Theta^A \\
\& [\Xi \Phi^{A,B,D}, A = \Delta \Omega(\Phi^{A,B,D}, A), A] \\
\lor & \hspace{1em} \Upsilon(\Xi \Phi^{A,B,D}, A, A) \vdash W^A \\
\lor & \hspace{1em} \overline{\Upsilon}(\Xi \Phi^{A,B,D}, A, A) \vdash \overline{W}^A
\end{align*}$$

Note — If $X \vdash W^A$ for each $A$-c.e. $W^A$, then $X \oplus A$ is 1-generic over $A$. 
Corresponding picture:

- Write $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{P}'$, $\mathcal{Q}'$, respectively, for $\mathcal{P}_\Theta$, $Q_{\Psi, \Phi, \hat{\Phi}, \Xi, \overline{W}, \overline{W}'}$, $\mathcal{P}_{\Theta'}$, $Q_{\Psi', \Phi', \hat{\Phi}', \Xi', \overline{W'}, \overline{W}'}$.

- Write $\mathcal{R}$ for the last 3 lines of $Q_{\Psi, \Phi, \hat{\Phi}, \Xi, \overline{W}, \overline{W}}$ above — its auxiliary clause — and $\mathcal{S}$ for the final disjunct of $\mathcal{R}$. 
Module for $\mathcal{P}$ below $\mathcal{Q}$:

Notes:

(a) **Define**: $\mathcal{Q}$-expansionary stage to be a stage at which $\ell(D, \Psi(\Phi_{A,B,D}, \hat{\Phi}_{A,B,D}, A))$ — the length of agreement of the arguments of $\ell$ — reaches a new maximum $> \text{any existing parameter observed at } \mathcal{Q}$.

(b) **Assume** that the module only acts at $\mathcal{Q}$-expansionary stages — so ignoring outcomes other than those infinitary ones in which $\mathcal{Q}$ implements the $\Gamma$ or $\Lambda$ strategies — and perhaps the $\Delta$ or $\Upsilon/\overline{\Upsilon}$ strategies — above $\mathcal{P}$.

(c) A number is *new* if it is greater than any number previously referred to.

(d) At the end of each stage $\Gamma_{\Phi_{A,B,D}, A}$, $\Lambda_{\Phi_{A,B,D}, A}$, $\Delta_{\Phi_{A,B,D}, A}$, $\Upsilon(\Xi_{\Phi_{A,B,D}, A}, A)$ and $\overline{\Upsilon}(\Xi_{\Phi_{A,B,D}, A}, A)$ are defined on an initial segment of the numbers.
The phases of the module:

1. Select a new $y$ as $D$-agitator for $\mathcal{P}$ (at $Q$).

2. Select a new $x$ to follow $\mathcal{P}$ — so $x > \varphi(\psi(y))$ and $> \hat{\varphi}(\psi(y))$,
   - designate the current values of $\Phi^{A,B,D} \upharpoonright \psi(y)$ and $\hat{\Phi}^{A,B,D} \upharpoonright \psi(y)$ their rectifying values, and
   - agitate via $y \searrow D$.

3. Ask:
   - Is it (a) $\Phi^{A,B,D} \upharpoonright \psi(y)$, or (b) $\hat{\Phi}^{A,B,D} \upharpoonright \psi(y)$, which has diverged from its rectifying value?

   - For (a), define $\Gamma_{\Phi^{A,B,D},A}(x) = B(x)$ with $\gamma(x)$ new $> \psi(y)$ ($x$ suits $\Gamma$), and otherwise —

   - For (b) — Define $\Gamma_{\Phi^{A,B,D},A}(x) = B(x)$ and $\Lambda_{\hat{\Phi}^{A,B,D},A}(x) = B(x)$ — with $\gamma(x)$ new $> \max \{\varphi(\psi(y)), \hat{\varphi}(\psi(y))\}$ and $\lambda(x)$ new $> \psi(y)$ ($x$ suits $\Lambda$).

4. Wait for $\ell(B, \Theta^A) > x$. 
5. Then check whether either (i) $x$ suits $\Gamma$ or (ii) $x$ suits $\Lambda$ and $\gamma(x) > \psi(x)$.

- If so, in either case, define $B(x) \neq \Theta^A(x)$ — let $y \not\in D$ — and restrain $A \upharpoonright \psi(x)$ and $A, B, D \upharpoonright \max \{ \varphi(\psi(y)), \hat{\varphi}(\psi(y)), \psi(y) \}$.

- And in case (ii), rectify $\Gamma$ at $x$ with an $A \upharpoonright \gamma(x)$ change.

**Outcome:** $P$ is satisfied, and in case (i) $\Gamma$ is rectified at $x$ via a return of $\Phi^{A,B,D} \upharpoonright \psi(y)$ to its rectifying value.

6. Otherwise:

- Change $A \upharpoonright \gamma(x)$ via $\gamma(x) \searrow A$, and return to 3 for $x$ — and initiate a new cycle via 1 (needed for the $\Lambda$ strategy).
Relating the strategies to the outcomes:

1. Select a new $D$-agitator $y$ for $\mathcal{P}$

2. Select a new follower $x$ for $\mathcal{P}$
   - Note the rectifying values of $\Phi^{A,B,D} \uparrow \psi(y)$ and $\hat{\Phi}^{A,B,D} \uparrow \psi(y)$
   - Agitate via $y \searrow D$

3. Is it (a) $\Phi^{A,B,D} \uparrow \psi(y)$ or (b) $\hat{\Phi}^{A,B,D} \uparrow \psi(y)$ differing from its rectifying value?
   - (a) Define $\Gamma^{\Phi^{A,B,D},A}(x) = B(x)$, $\gamma(x)$ new $\rightarrow x$ suits $\Gamma$
   - (b) Define $\Gamma^{\Phi^{A,B,D},A}(x) = B(x)$ and $A^{\hat{\Phi}^{A,B,D},A}(x) = B(x)$, $\gamma(x)$, $\lambda(x)$ new $\rightarrow x$ suits $\Lambda$

4. Wait for $\ell(B, \Theta^A) > x$

5. Does either (i) $\Gamma$ suit $x$, or (ii) $\Lambda$ and $\gamma(x) > \hat{\theta}(x)$?
   - 5. (cont.) Yes:
     - Define $B(x) \neq \Theta^A(x)$ and $y \not\nearrow D$
     - Restrain $A, B, D \uparrow \max \{\varphi(\psi(y)), \hat{\varphi}(\psi(y)), \psi(y)\}$ and $A \uparrow \hat{\theta}(x)$

   And $\gamma(x) \searrow A$ to rectify $\Gamma$
Analysis of outcomes:

$w_1$: Only finitely many $\mathcal{Q}$-expansionary stages.

**Outcome:** $\mathcal{Q}$ becomes trivially satisfied via $D \neq \Psi(\Phi^{A,B,D}, \widehat{\Phi}^{A,B,D}, A)$ — and offers no obstacle to the eventual satisfaction of $\mathcal{P}$.

- *Otherwise:* The module cannot terminate at any of phases 1, 2 or 3 — in the case of 3, either (a) or (b) being guaranteed by the choice of $\gamma(x)$ in 3.

$w_2$: Module terminates at 4.

**Outcome:** $\mathcal{P}$ trivially satisfied, and provides no further impediment to the strategies for $\mathcal{Q}$.

$s_1$: The module terminates at 5.

**Outcome:** $\mathcal{P}$ is satisfied via $\Theta^A(x) \downarrow \neq B(x)$.

- And in subcase (i) — as observed above — $\Gamma$ is rectified at $x$ via a return of $\Phi^{A,B,D} \uparrow \psi(y)$ to its rectifying value. Where —
• This return is ensured by the return of $D(y)$ to its value at the start of phase 2, and by the conservation of $A, B, D \uparrow \varphi(\psi(y))$ following 2 — these uses being intact following phase 5, by the choice of $x > \varphi(\psi(y))$ in 2 — and also following any intervening visit to phase 6 by the newness of any $\gamma(x)$ appointed via 3.

• While the sufficiency of this return for the rectification of $\Gamma$ at $x$ is guaranteed by the newness of $x$ in 2 — so that $\Gamma^{\Phi^{A,B,D},A}(x) \uparrow$ at all stages previous to that at which 3 is implemented in relation to $x$, and by the maintenance of this situation until 3 applies by $y \smallsetminus D$ in 2 —

• By which time $x$ suits $\Gamma$, and one defines $\gamma(x)$ new, and so $> \text{ any previous value of } \psi(y)$ — including that pertaining to the rectifying value of $\Phi^{A,B,D} \uparrow \psi(y)$.  

Note: In this simple module, the newness of $\gamma(x)$ in 3 precludes any switch from (a) to (b) — or vice versa — consequent on repeated visits to 3 on behalf of $x$. 
\( i_1 \): Infinitely many applications of phase 6.

- One first notices that in this case \( \mathcal{P} \) is satisfied via \( \Theta^A(x) \uparrow \) — since each time 5 fails, one must have \( \vartheta(x) \geq \) the current value of \( \gamma(x) \) — which is renewed on each return to 3 in relation to \( x \).

- Also involves \( \Gamma^{\Phi^{A,B,D},A}(x) \uparrow \) in the limit —

- \textbf{But} one can verify that the \( \Lambda \) strategy for \( \mathcal{Q} \) — elaborated below in relation to the other \( \mathcal{P} \) requirements — has been progressed.

- This is because — just as for \( \Gamma \) in subcase (i) of outcome \( s_1 \) above — one can verify that the timing of the definition of \( \Lambda^{\Phi^{A,B,D},A}(x) \) and the conservations of uses implicit in the implementation of the module ensure that \( \Lambda \) is rectifiable at \( x \) via any return to the rectifying value of \( \Phi^{A,B,D} \uparrow \psi(y) \).

- The initiation of new cycles following 6 will provide infinitely many such numbers \( x \) for the \( \Lambda \) strategy.
**Note:** The satisfaction of \( P \) is not essential to outcome \( i_1 \) — since that may also be accomplished via a lower priority copy of \( P \) within the context of the \( \Lambda \) strategy for \( Q \) — But then phase 6 is integral to the outcome \( \hat{i}_1 \) below.

**The module for \( P' \) below \( P \) below \( Q \):**

**Note:** Only need to examine the module for \( P \) in the cases of either —

(a) Finitary \( P \) or \( P' \) activity disrupting \( P' \)- or \( P \)- led rectification of \( \Gamma \) or \( \Lambda \), or —

(b) Infinitary activity on \( P \) — leading on to implementation of the \( \Lambda \) strategy for \( Q \) above \( P' \).
Case (a): $\mathcal{P}'$ implements the $\Gamma$ strategy for $\mathcal{Q}$ (according to the previous module).

- One needs to examine the various modes of $\mathcal{P}$ activity in relation to the possible $\mathcal{P}'$ activity, according to their potential for mutual disruption of rectifying strategies.
- The only relevant $\mathcal{P}'$ activity is $\mathcal{P}'$ satisfaction via 5, or finitary implementation of 6 by $\mathcal{P}'$.
- And the only relevant $\mathcal{P}$ activity is $D$-agitation via 2 — implementation of a $B(x)$ change via 5 — or of an $A(\gamma(x))$ change via 6.

- **Need to check** that in this case the module is obtained by allowing a module as above for each of $\mathcal{P}$ and $\mathcal{P}'$ —
- With the following rule for conjoining —
Whenever the module for $\mathcal{P}$ applies — implement it.

While initialising that for $\mathcal{P}'$ — by cancelling any existing $\mathcal{P}'$ related $D$-agitator $y'$ or follower $x'$, and — if needed — rectifying $\Gamma$ at $x'$ by enumerating $\gamma(x') \setminus A$ — and initialising any $\Lambda$ being built by $\mathcal{P}'$ by removing all its current axioms.

Clearly — no unrectified $\Gamma$ or $\Lambda$ remains following an application of the $\mathcal{P}$ module — so long as $\gamma(x') \setminus A$ does not come into conflict with the conservations of uses on which the $\mathcal{P}$ module depends.

And for the latter — one needs to examine the phases of the module for $\mathcal{P}$ — while noting that these are only accompanied by $\Gamma$ rectification at $x'$ if $\mathcal{P}'$ is currently satisfied via a previous terminal application of 5 for $\mathcal{P}'$ —

And that such rectification is only a problem for $\mathcal{P}$ if it ignores a current conservation needed at $\mathcal{P}$.
• But if $\mathcal{P}'$ is satisfied, and $y$ already exists — one must have $y' > y$, and — by the description of 2 and 3 for $\mathcal{P}'$, and the corresponding conservations at $\mathcal{P}'$ —

$$\gamma(x') > \max \{ \varphi(\psi(y')), \hat{\varphi}(\psi(y')), \psi(y') \}$$

$$\geq \max \{ \varphi(\psi(y)), \hat{\varphi}(\psi(y)), \psi(y) \}$$

when $\mathcal{P}$ requires attention.

Case (b): $\mathcal{P}'$ implements the $\Lambda$ strategy for $\mathcal{Q}$ (with $\Lambda$ being built by $\mathcal{P}$).

• Here one assumes $\mathcal{P}$ implementing its module as before.

•• But in the case of infinitely many returns to 1 on behalf of $(\mathcal{P}, \mathcal{Q})$ — one needs to describe an auxiliary strategy for $\Lambda = \Lambda_{\mathcal{P}}$ — which synchronises its activities with those of the module for $\mathcal{P}$ working relative to $\Gamma$ —
1. Select a new $y'$ as $D$-agitator for $\mathcal{P}'$ (at $Q$), with $y' < y$.

2. Select a new $x' > x$ to follow $\mathcal{P}'$ (so $x' > \varphi(\psi(y))$ and $> \hat{\varphi}(\psi(y))$), let the current value of $\hat{\Phi}^{A,B,D} \upharpoonright \psi(y')$ be its rectifying value, and agitate via $y' \searrow D$.

3. Define $\Lambda_{\hat{\Phi}^{A,B,D}}(x') = B(x')$, with $\lambda(x')$ new $> \max \{ \varphi(\psi(y')), \hat{\varphi}(\psi(y')), \psi(y') \}$.

4. Wait for $\ell(B, \Theta'^A) > x'$.

5. Assume that 5(i) and (ii) do not apply (otherwise the satisfaction of $\mathcal{P}$, and the consequent termination of the $\Lambda_{\mathcal{P}}$ strategy, is accompanied by $y \nearrow D$ and $y' \nearrow D$).
   
   • Check if $\gamma(x) > \vartheta'(x')$.
   
   • If so, define $B(x') \neq \Theta'^A(x')$, let $y, y' \nearrow D$, restrain $A, B, D \upharpoonright \max \{ \varphi(\psi(y)), \hat{\varphi}(\psi(y)), \psi(y) \}$ and $A \upharpoonright \vartheta'(x')$ — while $\mathcal{P}$ must return to 1. $\mathcal{P}'$ is satisfied.

6. Otherwise: Return to 4 to await an opportunity to accompany $\mathcal{P}$ to 5/5.
Relating the strategies to the outcomes:

1. Select a new $D$-agitator $y$ for $\mathcal{P}$

2. Select a new follower $x$ for $\mathcal{P}$
   - Note the rectifying values of $\Phi^y A, B, D | \psi(y)$ and $\hat{\Phi}^y A, B, D | \hat{\psi}(y)$
   - Agitate via $y \setminus D$

3. Is it (a) $\Phi^y A, B, D | \psi(y)$ or (b) $\hat{\Phi}^y A, B, D | \hat{\psi}(y)$ differing from its rectifying value?

   (a) Define $\Gamma^{\Phi^y A, B, D, A}(x) = B(x)$, $\gamma(x)$ new $\rightarrow x$ suits $\Gamma$

   (b) Define $\Gamma^{\hat{\Phi}^y A, B, D, A}(x) = B(x)$ and $\Lambda^{\hat{\Phi}^y A, B, D, A}(x) = B(x)$, $\gamma(x)$, $\lambda(x)$ new $\rightarrow x$ suits $\Lambda$

4. Wait for $\ell(B, \Theta^A) > x$

5. Does either (i) $x$ suit $\Gamma$, or (ii) $x$ suit $\Lambda$ and $\gamma(x) > \vartheta(x)$?

5. (cont.) Yes:
   - Define $B(x) \neq \Theta^A(x)$ and $y \not\sim D$
   - Restrain $A, B, D \upharpoonright \max \{\varphi(\psi(y)), \hat{\varphi}(\hat{\psi}(y)), \psi(y)\}$ and $A \upharpoonright \vartheta(x)$

   And $y' \not\sim D$

6. No: $\gamma(x) \setminus A$

   Yes: Define $B(x') \neq \Theta^A(x')$, $y, y' \not\sim D$, restrain $A, B, D \upharpoonright \max \{\varphi(\psi(y)), \hat{\varphi}(\hat{\psi}(y)), \psi(y)\}$ and $A \upharpoonright \vartheta'(x')$

   And $\gamma(x) \setminus A$ to rectify $\Gamma$
Note: Will see later that more requirements $\mathcal{P}'$ will potentially force infinitely many returns to $1$ — Pairs $(x', y')$ being paired individually with pairs $(x, y)$, the order of appointment following the priority of $(x', y')$.

Analysis of outcomes, including those for the auxiliary strategy (for case (b)).

$\boxed{w_1}$: Only finitely many $Q$-expansionary stages — as before giving trivial satisfaction of $Q$, which ceases to interfere with $\mathcal{P}$ and $\mathcal{P}'$.

$\boxed{w_2} / \boxed{\hat{w}_2}$: If either $\mathcal{P}$ or $\mathcal{P}'$ halts at $4$ or $\hat{4}$, respectively, then it is trivially satisfied — and removed from the subsequent environment of the other requirements.

$\boxed{s_1} / \boxed{\hat{s}_1}$: Implementation of $5/\hat{5}$, satisfying $\mathcal{P}$ or $\mathcal{P}'$, respectively.
Need to verify: In each case the $\Gamma/\Lambda$ strategies remain intact.

- Updating the analysis for case (a), one can see that one only needs to consider satisfaction of $P'$ via $\hat{5}$ —

- This is because the above analysis for $s_1$ still holds —

- Since if $\hat{2}$, with $y' \not\xrightarrow{} D$, accompanies the implementation of 2 and $y \not\xrightarrow{} D$ — then any implementation of 5(i) or (ii) with $y \xrightarrow{} D$ is accompanied by $y' \xrightarrow{} D$.

- So following 5(i) $\Gamma$ is again rectified at $x$ via a return of $\Phi^{A,B,D} \upharpoonright \psi(y)$ to its rectifying value.

- The conservations on $A, B, D \upharpoonright \varphi(\psi(y))$ needed following 2 are still intact following the augmented phase 5, by the choice of $x' > x > \varphi(\psi(y))$ in $2/\hat{2}$ —
• And the sufficiency of the return to the rectifying value for the rectification of $\Gamma$ at $x$ is guaranteed as before.

• And as previously, the $\Gamma$ strategy remains intact in subcase (ii), due to the rectification of $\Gamma$ at $x$ via an $A\models \gamma(x)$ change.

• Say now $\mathcal{P}'$ is satisfied via $\hat{5}$.

• Then since an instance of $\hat{5}$ can only accompany one of 5 leading to 6 for $\mathcal{P}$ — outcome $\hat{s}_1$ must involve $\gamma(x) \smallsetminus A$ — and since $x' > x$ — giving $\gamma(x') \geq \gamma(x)$ — consequent $\Gamma$ rectification at $x'$.

• On the other hand — similarly to the argument for $\Gamma$ in case 5(i) of $s_1$ — one can verify that $\Lambda$ is rectified at $x'$ by a return to the rectifying value of $\hat{\Phi}^{A,B,D} | \psi(y')$ —
• Otherwise, since $y' < y$, one cannot have a return to the rectifying value of $\Phi^{A,B,D} \upharpoonright \psi(y)$ following $\hat{5}$.

• This could only happen via some injury to the conservations at $\mathcal{P}$ other than through $y, y' \nrightarrow D$ — Which is impossible since have $x', \gamma(x) > \max \{\varphi(\psi(y)), \hat{\varphi}(\psi(y))\}$.

(For a closer examination of how this situation is maintained following multiple implementations of $6/\hat{6}$, see below.)

$i_1$: Infinitely many applications of phase 6.

• The analysis as it relates to $\mathcal{P}$ below $\mathcal{Q}$ is as before.

• Only remains to elaborate on the remarks concerning the progression of the $\Lambda$ strategy for $\mathcal{Q}$ — along the lines of those concerning $\Gamma$ in subcase (i) of outcome $s_1$ above.
• One needs to be sure, following an implementation of 6/\hat{6}, that \Lambda rectification at \( x' \) is still guaranteed by the return to the rectifying value of \( \hat{\Phi}^{A,B,D} \upharpoonright \psi(y') \) following any subsequent satisfaction of \( \mathcal{P} \) via \( \hat{5} \).

• And this holds so long as no axiom for \( \Lambda \) is enumerated with use \( \hat{\Phi}^{A,B,D} \upharpoonright \lambda(x') \supseteq \) the rectifying value for \( \hat{\Phi}^{A,B,D} \upharpoonright \psi(y') \) and value 0 at argument \( x' \).

• Assume otherwise.

• So at some stage \( t \) following the determination of the rectifying value for \( \hat{\Phi}^{A,B,D} \upharpoonright \psi(y') \) via \( \hat{2} \) — at a stage \( s_0 \), say — and before the enumeration of \( x' \) into \( B \) via \( \hat{5} \) — at a stage \( s_1 \) say > \( s_0 \) — one defines such an axiom for \( \Lambda \) via \( \hat{3} \) of the form \( \Lambda(\hat{\Phi}^{A,B,D} \upharpoonright \lambda(x'), A \upharpoonright \lambda(x'), x') = 0 \).

• But at each such stage \( t, s_0 < t < s_1 \) one has \( y, y' \in D \) —
• And hence either —

• \( \Phi^{A,B,D}[t] \) is incompatible with the rectifying values of \( \Phi^{A,B,D} \upharpoonright \psi(y), \ \Phi^{A,B,D} \upharpoonright \psi(y') \), or —

• \( \hat{\Phi}^{A,B,D}[t] \) is incompatible with the rectifying values of \( \hat{\Phi}^{A,B,D} \upharpoonright \psi(y), \ \hat{\Phi}^{A,B,D} \upharpoonright \psi(y') \), or —

• \( A[t] \) is incompatible with both \( A \upharpoonright \psi(y)[s_0] \), \( A \upharpoonright \psi(y')[s_0] \).

• The last case cannot occur — since such an \( A \)-change can only happen via 6 (\( \mathcal{P} \) is not satisfied via 5(ii) at any stage) — and \( \gamma(x) \) is always selected via 3 to be new \( > \psi(y) \).

• And if the first alternative applies at some such stage — there must at some later stage before \( s_1 \) be a switch away from \( x \) suiting \( \Gamma \) — to preclude satisfaction of \( \mathcal{P} \) via 5(i).

• Again, this can only happen via \( \gamma(x) \ \downarrow \ A \) — impossible previous to 5.
Finally, no such axiom for $\Lambda$ can be defined at a stage $t$ at which $\hat{\Phi}^{A,B,D}[t]$ is incompatible with the rectifying value of $\hat{\Phi}^{A,B,D} | \psi(y')$.

$i_1$: Infinitely many applications of phase $\hat{6}$.

**Outcome:** $\mathcal{P}'$ is satisfied via $\Theta'^A(x') \uparrow$, and — as for $i_1$ above — one has an intact $\Lambda$ strategy for $\mathcal{Q}$ to pass on to any other $\mathcal{P}$-requirements below $\mathcal{P}$.

- **The following sequence of modules** will cumulatively comprise the full module.
- In each case, the corresponding successful outcomes will be outlined immediately after each module — and shown not to injure those for previously described modules.
- The priority ordering of the relevant requirements — together with a preliminary indication of their roles in relation to the strategies — is given by —
Level 1: Module for $\mathcal{P}$ below $\mathcal{Q}$ below $\mathcal{Q}'$ ($\mathcal{P}$ implementing the $\Gamma$ and $\Gamma'$ strategies):

- All modules act at $\mathcal{Q}'$-expansionary stages — and — unless otherwise stated — at $\mathcal{Q}$-expansionary stages (as observed relative to the $\mathcal{Q}'$-expansionary stages).

- All modules below operate in the context of activity related to a successful $\Gamma'$ strategy — but allow for implementations of a $\Lambda'$ strategy off the eventual true path — with accompanying $\Gamma'$ rectification via $A$-changes.

At level 1, a requirement $\mathcal{P}$ implements the following phases —
$1^\Gamma / 1^{\Gamma'}$. Select a new $y$ as $D$-agitator for $\mathcal{P}$ (at $\mathcal{Q}$ and $\mathcal{Q}'$).

$2^\Gamma / 2^{\Gamma'}$. Select a new $x$ to follow $\mathcal{P}$ — so that $x > \text{each of } \varphi'(\psi'(y))$, $\hat{\varphi}'(\psi'(y))$, $\varphi(\psi(y))$ and $\hat{\varphi}(\psi(y))$. Then —

- Designate the current values of $\Phi^{A,B,D} \upharpoonright \psi(y)$, $\Phi'^{A,B,D} \upharpoonright \psi'(y)$, $\hat{\Phi}^{A,B,D} \upharpoonright \psi(y)$ and $\hat{\Phi}'^{A,B,D} \upharpoonright \psi'(y)$ their rectifying values, and

- Agitate via $y \setminus D$.

- Proceed to $3^{\Gamma'}$, and then $3^\Gamma$.

$3^{\Gamma'}$. ($\mathcal{Q}'$-expansionary)

- **Ask:** Is it (a) $\Phi'^{A,B,D} \upharpoonright \psi'(y)$ ($x$ suits $\Gamma'$) — or (b) $\hat{\Phi}'^{A,B,D} \upharpoonright \psi'(y)$ ($x$ suits $\Lambda'$) — which has diverged from its rectifying value?

- In either case, define $\Gamma'\Phi'^{A,B,D}$, $A(x) = B(x)$ with $\gamma'(x)$ new (so $\gamma'(x) > \text{any } \psi'(y)$ so far, and $> \max \{ \varphi'(\psi'(y)), \hat{\varphi}'(\psi'(y)) \}$) —

- And proceed to $3^\Gamma$. 
3\(\Gamma\). (Similar to 3\(\Gamma'\).)

- **Ask:** Is (a) \(\Phi^{A,B,D} \upharpoonright \psi(y) \ (x \text{ suits } \Gamma)\) — or (b) \(\check{\Phi}^{A,B,D} \upharpoonright \psi(y) \ (x \text{ suits } \Lambda)\) — incompatible with its rectifying value?

- In either case, define \(\Gamma^{\Phi^{A,B,D},A}(x) = B(x)\) with \(\gamma(x)\) new > \(\gamma'(x)\) and > \(\psi(y)\) — and proceed to 4\(\Gamma\) / 4\(\Gamma'\).

4\(\Gamma\) / 4\(\Gamma'\). *Wait* for a \(\mathcal{P}\)-expansionary stage — that is, one at which \(\ell(B, \Theta^A) > \) each current parameter — and, in particular, > \(x\).

5\(\Gamma\) / 5\(\Gamma'\). *Check* whether either (i) \(x \text{ suits } \Gamma'\), or (ii) \(x \text{ suits } \Lambda'\) and \(\gamma'(x) > \vartheta(x)\) —

- *And* whether either (iii) \(x \text{ suits } \Gamma\), or (iv) if (i) holds and \(x \text{ suits } \Lambda\), then \(\gamma(x) > \vartheta(x)\).

- **If so** — In each case, let \(y \not\rightarrow D\), define \(B(x) \neq \Theta^A(x)\) — and restrain \(A \upharpoonright \vartheta(x)\) and \(A, B, D \upharpoonright \max \{\varphi'(\psi'(y)), \check{\varphi}'(\psi'(y)), \psi'(y)\}\).
• In cases (ii) or (iv), rectify $\Gamma'$, $\Gamma$ at $x$ via an $A \upharpoonright \gamma'(x)$ change, or via an $A \upharpoonright \gamma(x)$ change, respectively.

• $\mathcal{P}$ is now satisfied within the $\Gamma'$ and $\Gamma$ strategies for $\mathcal{Q}'$ and $\mathcal{Q}$ — and the $\Lambda' = \Lambda'_{\mathcal{P}}$ and $\Lambda = \Lambda_{\mathcal{P}}$ strategies for $\mathcal{Q}'$ and $\mathcal{Q}$ are initialised.

• If none of these alternatives applies — proceed to $6^{\Gamma'}$ if (i) and (ii) fail — and otherwise to $6^{\Gamma}$ if both (iii) and (iv) fail.

$6^{\Gamma'} / 6^{\Gamma}$. Change $A \upharpoonright \gamma'(x)$ or $A \upharpoonright \gamma(x)$ via $\gamma'(x) \downharpoonright A$ or $\gamma(x) \downharpoonright A$, respectively — and return to $3^{\Gamma'}$ for $x$.

• Initiate a new cycle via $1^{\Gamma} / 1^{\Gamma'}$ — in anticipation of a failure to satisfy $\mathcal{P}$ within the $\Gamma'$ and $\Gamma$ strategies using $x$ —

• And progress the $\Lambda'$ or $\Lambda$ strategy correspondingly — anticipating eventual failure to satisfy $\mathcal{P}$ within the $\Gamma'$ and $\Gamma$ strategies using any follower of $\mathcal{P}$.
Analysis of outcomes:

- It is easy to see that the possible outcomes relative to $Q'$ are as previously —
- And that the only ones extending those relative to $Q$ involve a valid $\Gamma'$ strategy prevented from facilitating finitary satisfaction of $P$ by $Q$.

$[w_1'] / [w_1]$ : Only finitely many $Q'$-expansionary stages, or $Q$-expansionary stages, respectively. $Q'$ or $Q$, respectively, satisfied as before, with consequent reduction to a preceding module.

$[w_2]$ : $P$ halts at $4^\Gamma / 4^{\Gamma'}$. Then $P$ is trivially satisfied, as before.

$s_1$ : Implementation of $5^\Gamma / 5^{\Gamma'}$, satisfying $P$.

- With regard to the verification that the $\Gamma'/\Lambda'$ and $\Gamma/\Lambda$ strategies remain intact — one first observes that $P$-satisfaction via $5^\Gamma / 5^{\Gamma'}$ involves initiation of the $\Lambda_P$ and $\Lambda'_P$ strategies.
• If possibilities (i) and (iii) applied in $5\Gamma / 5\Gamma'$, then one needs to check that $\Gamma'$ and $\Gamma$ are rectified at $x$.

• Arguing as before — It is straightforward to see that at no stage does an application of phase $6\Gamma / 6\Gamma'$ result, for a given $x$ — remembering that $\gamma(x) > \gamma'(x)$ — in a switch from subcase (a) to subcase (b), or vice versa, in successive applications of phase $3\Gamma'$.

• And so — arguing as before in this case — $\Gamma'$ is rectified at $x$ in $5\Gamma / 5\Gamma'$ via a return to the rectifying value of $\Phi^{A,B,D}_{\psi'}(y)$.

• Also — It is easy to see that one can only get such a change in phase $3\Gamma$ via an occurrence of phase $6\Gamma'$ in relation to $x$ — in which case $\Gamma$ is subsequently rectified via $\gamma'(x) \searrow A$ (since $\gamma(x) > \gamma'(x)$).

• This means —
• If the eventual outcome is satisfaction of $\mathcal{P}$ via an application of $5^\Gamma / 5^{\Gamma'}$ with subcase (iii) applying, then — arguing as before — one can verify that $\Gamma$ is rectified at $x$ in $5^\Gamma / 5^{\Gamma'}$ via a return to the rectifying value of $\Phi'^{A,B,D} \uparrow \psi'(y')$ (since $y' < y$).

•• If cases (i) and (iv) applied in $5^\Gamma / 5^{\Gamma'}$ — $\Gamma'$ is rectified at $x$ via a return to the rectifying value of $\Phi'^{A,B,D} \uparrow \psi'(y')$, as before — and $\Gamma$ is also rectified via $\gamma(x) \setminus A$.

•• In case (ii) of $5^\Gamma / 5^{\Gamma'}$ — $\Gamma'$ is rectified at $x$ via $\gamma'(x) \setminus A$ —

• And in case (iv), $\Gamma$ is similarly rectified.

•• Finally — In case (iii) one can argue as previously that $\Gamma$ is rectified at $x$ via a return to the rectifying value of $\Phi^{A,B,D} \uparrow \psi(y')$ — since the prior intervention of phase $6^{\Gamma'}$ involves the rectification of $\Gamma$ via $\gamma'(x) \setminus A$. 
\([i_1'] / [i_1]\): Infinitely many applications of phase \(6^{\Gamma'}\) or \(6^\Gamma\).

- In either case \(\mathcal{P}\) is satisfied via \(\Theta^A(x)^\uparrow\), as before —

- Although — since this is not within the \(\Gamma\) and \(\Gamma'\) strategies — the incorporation of further \(\mathcal{P}\)-requirements will entail verification of the provision of valid \(\Lambda'\) or \(\Lambda\) strategies for dealing with \(\mathcal{P}\) in a more general setting.

- (In the case of \([i_1]\) but not outcome \([i_1']\), one notices that the \(\Gamma'\) strategy does remain intact.)

- **In the case of infinitely many returns to phase \(1^\Gamma / 1^{\Gamma'}\), need —**
Level 2: Module for $\hat{P}$ below $P$, below $Q$ below $Q'$ (with $\hat{P}$ applying the $\Lambda$ or $\Lambda'$ strategy):

- Assume that infinitely many followers for $P$ are selected via $1^\Gamma / 1^{\Gamma'}$.

- The auxiliary strategies will be built, as required, by $P$ — and will be applied, as appropriate, by suitable copies of $\hat{P}$.

- The phases below synchronise their activities with those of the corresponding phases of the preceding module.

$1^\Lambda / 1^{\Lambda'}$. Select a new $\hat{y}$ as $D$-agitator for $\hat{P}$ (at $Q$ and $Q'$), with $\hat{y} < y$.

$2^\Lambda / 2^{\Lambda'}$. Select a new $\hat{x} > x$ to follow $\hat{P}$ — so $\hat{x} > \varphi'(\psi'(y)), \varphi'(\psi'(y)), \varphi(\psi(y))$ and $\varphi(\psi(y))$.

- Let the current value of $\hat{\Phi}_A^{A,B,D} \uparrow \psi(\hat{y})$ be its rectifying value — And agitate via $\hat{y} \downarrow D$. 

\[ 3^{\Lambda'} \] (Simultaneous with \( 3^{\Gamma'} \) (b) applying).

- Take \( \lambda'(\hat{x}) \) new, greater than \( \max \{ \varphi'(\psi'(\hat{y})), \hat{\varphi}'(\psi'(\hat{y})), \psi'(\hat{y}) \} \).

\[ 3^{\Lambda} \] (Simultaneous with \( 3^{\Gamma} \) (b) applying).

- Define \( \Lambda^{\hat{\Phi}^{A,B,D},A}(\hat{x}) = B(\hat{x}) \) — with \( \lambda(\hat{x}) \) new > \( \max \{ \varphi(\psi(\hat{y})), \hat{\varphi}(\psi(\hat{y})), \psi(\hat{y}) \} \).

\( 4^{\Lambda} / 4^{\Lambda'}. \) Wait for a \( \hat{\mathcal{P}} \)-expansionary stage.

\( 5^{\Lambda'}. \) Ask if either (i') (i) or (ii) of \( 5^{\Gamma} / 5^{\Gamma'} \) hold, or otherwise whether (i'') \( \gamma'(\hat{x}) > \hat{\nu}(\hat{x}) \), or not.

- Yes — Proceed to \( 5^{\Lambda} \), while in subcase (i') — initialise \( \Lambda' \).

- No — Proceed to \( 6^{\Lambda'}. \)

\( 5^{\Lambda}. \) Ask if either (i) (iii) or (iv) of \( 5^{\Gamma} / 5^{\Gamma'} \) hold, or otherwise whether — (i"") (i"") the rectifying value of \( \hat{\Phi}^{A,B,D}\upharpoonright \psi(\hat{y}) \) suffices to rectify \( \Lambda \) at \( \hat{x} \), or (i'""") \( \lambda(\hat{x}) > \hat{\nu}(\hat{x}) \).

- If so —
• In each case — Let \( y, \hat{y} \not\in D \), define \( B(\hat{x}) \neq \hat{\Theta}^A(\hat{x}) \) — and restrain \( A\upharpoonright \hat{\vartheta}(\hat{x}) \) and \( A, B, D\upharpoonright \max \{ \varphi'(\psi'(\hat{y})), \hat{\varphi}'(\psi'(\hat{y})), \psi'(\hat{y}) \} \).

• In the case that (\( i \)) and \( 5^\Lambda' \) subcase (\( i' \)) hold — Initialise the \( \Lambda \) and \( \Lambda' \) strategies.

• As before — \( P \) is satisfied within the \( \Gamma / \Gamma' \) strategies.

• \textit{Rectify} \( \Gamma \) at \( \hat{x} \) via an \( A\upharpoonright \gamma(\hat{x}) \) change — And in case (\( iv \)) rectify \( \Lambda \) at \( \hat{x} \) via an \( A\upharpoonright \lambda(\hat{x}) \) change.

• \( \hat{P} \) is now \textit{satisfied} within the \( \Lambda / \Gamma \) strategy (according as \( 5^\Lambda(\hat{i}) \) applied or not) — and within the \( \Lambda' / \Gamma' \) strategy (according as \( 5^\Lambda'(\hat{i}') \) applied or not).

• If none of these alternatives applies —

• \textit{Proceed} to \( 6^\Lambda \), when \textit{activated} by the \( \Gamma / \Lambda \) strategy for \( R \).
\[6^\Lambda \quad (\Gamma/\Lambda \text{ breakdown}). \quad \text{Change } A \uparrow \lambda(\hat{x}) \text{ via } \lambda(\hat{x}) \searrow A.\]

- And if \( R \) is not already satisfied —
- \textit{Return} to \( 3^{\Lambda'} \) for \( \hat{x} \) — and progress the \( \Upsilon/\Delta \) strategy for \( R \) (see below).

\section*{Analysis of outcomes:

\subsection*{Impact on the level 1 outcomes:}

- On \( w_1, w'_1 \) and \( w_2 \) — None.
- On \( s_1 \) —
  - Since \( \hat{x} > x \), have \( \gamma(\hat{x}) > \gamma(x), \quad \gamma'(\hat{x}) > \gamma'(x). \)
  - Also \( \lambda'(\hat{x}), \quad \lambda(\hat{x}) \) are chosen to be new simultaneously with \( 3^\Gamma / 3^{\Gamma'} \).
- This means, as before — No application of \( 6^{\Lambda'} \) or \( 6^\Lambda \) can result in a switch from subcase (a) to subcase (b), or vice versa, in successive applications of phase \( 6^{\Gamma'} \) of the level 1 module.
• Also — the augmented agitation via $2^\Lambda / 2^{\Lambda'}$ is — simultaneously with $5^\Lambda / 5^{\Lambda'}$ — restored via $5^\Lambda$, in the case that $\mathcal{P}$ becomes satisfied via $5^\Lambda / 5^{\Lambda'}$.

• This means that in each case appropriate to $s_1$, the auxiliary strategies do not materially alter the relevant rectifications of $\Gamma$ and $\Gamma'$.

•• The infinitary outcomes $i_1$ and $i'_1$ are subsumed in those for requirements below $\mathcal{P}$ working within the context of valid auxiliary strategies.

**Outcomes specific to the level 2 strategies:**

$\hat{w}_2$: $\hat{\mathcal{P}}$ halts at $4^\Lambda / 4^{\Lambda'}$.

• Get $\hat{\mathcal{P}}$ trivially satisfied — similarly to $\mathcal{P}$ in the case of outcome $w_2$ at level 1.

$\hat{s}_1$: Implementation of $5^\Lambda$ satisfying $\hat{\mathcal{P}}$. 
Need to verify: In each case the $\Gamma/\Lambda$ and $\Gamma'/\Lambda'$ strategies remain intact.

• Say $\hat{\mathcal{P}}$ is satisfied via $5^\Lambda$.

• If such satisfaction is within the $\Lambda$ or $\Lambda'$ strategy, such an application of $5^\Lambda$ can only accompany one of $5^\Gamma$ leading to $6^\Gamma$ or $6^\Gamma'$, respectively, for $\mathcal{P}$.

• In which case $s_1$ will involve $\gamma(x) \searrow A$ or $\gamma'(x) \searrow A$, respectively — and since $x' > x$, giving $\gamma(x') \geq \gamma(x)$, a corresponding $\Gamma$ or $\Gamma'$ rectification at $\hat{x}$.

• On the other hand — if the satisfaction of $\hat{\mathcal{P}}$ is within the $\Gamma'$ strategy —

• The previous argument for $\Gamma$ in case $5^\Gamma(i)$ of $s_1$ is easily adaptable — and one can verify that $\Gamma'$ is rectified at $\hat{x}$ by a return to the rectifying value of $\Phi'^{A,B,D} | \psi'(\hat{y})$ —

• Otherwise — since $\hat{y} < y$ — one cannot have a return to the rectifying value of $\Phi'^{A,B,D} | \psi'(y)$ following $5^\Lambda$. 
• Since this can only happen via injury to this rectifying value other than through \( y, \hat{y} \not\in D \) — impossible since have \( \hat{x} \) and \( \gamma(x) > \max \{ \varphi'(\psi'(y)), \varphi'(\psi'(\hat{y})) \} \).

• While as previously — an examination of the effects of multiple implementations of \( 6^\Gamma / 6^{\Gamma'} \) shows that these cannot alter the fact that a return to the rectifying value of \( \Phi'_{A,B,D} \upharpoonright \psi'(\hat{y}) \) does lead to rectification of \( \Gamma' \) at \( \hat{x} \).

• A similar argument holds in the case of satisfaction of \( \hat{\mathcal{P}} \) is within the \( \Gamma \) strategy.

• On the other hand — one can similarly verify that if the satisfaction of \( \hat{\mathcal{P}} \) is within the \( \Lambda' \) strategy, \( \Lambda' \) is either rectified at \( \hat{x} \) by \( \lambda'(\hat{x}) \searrow A \) — or by a return to the rectifying value of \( \hat{\Phi'}_{A,B,D} \upharpoonright \psi'(\hat{y}) \).

• While if the satisfaction of \( \hat{\mathcal{P}} \) is within the \( \Lambda \) strategy — the conditions of \( 5^\Lambda \) ensure that \( \Lambda \) is either rectified at \( \hat{x} \) by \( \lambda(\hat{x}) \searrow A \) — or by a return to the rectifying value of \( \hat{\Phi}_{A,B,D} \upharpoonright \psi(\hat{y}) \).
$\hat{i}_1$: Infinitely many applications of phase $6^A$.

**Outcome:** $\hat{P}$ is satisfied via $\hat{\Theta}^A(\hat{x}) \uparrow$.

- While to replace the failed $\Gamma/\Lambda$ strategies — one needs to provide a valid $\Upsilon$ or $\Delta$ strategy for $Q$ to pass on to $P$-requirements below $\hat{P}$ —
Level 3: Module for $\tilde{\mathcal{P}}$ below $\mathcal{P}$ below $\mathcal{P}$, acting below $Q$ below $Q'$ ($\mathcal{P}$ implementing the $\Upsilon/\Delta$ strategy for $Q$):

- Assume that on infinitely many occasions $\mathcal{P}$ selects a new follower via $\hat{1}$.

The $\Upsilon/\Delta$ strategy for $\mathcal{R}$:

Note:

- $\tilde{\mathcal{P}}$ may become satisfied below within the $\Delta$ strategy provided at $\mathcal{R}$ — or may indirectly instigate the outright satisfaction of $\mathcal{R}$ resulting in the termination of this strategy.

- In either case the $\Upsilon$ strategy survives — and features in an infinite list of $\mathcal{R}$-subrequirements dispersed throughout the prioritised listing of requirements — while continuing to work with the $\Gamma/\Lambda$ strategy it depends on.
Background activity:

(a) $\Xi\widehat{\Phi}^{A,B,D,A}$-expansionary stages are ones at which one observes — during an implementation of the main module — an increase in the available beginnings of $\Xi\widehat{\Phi}^{A,B,D,A}$ — due to $\ell(\Xi\widehat{\Phi}^{A,B,D,A})$ reaching a new maximum > than any existing parameter actively associated with the strategy for $\mathcal{R}$.

(b) The $\mathcal{R}$-expansionary stages are those which are $\Xi\widehat{\Phi}^{A,B,D,A}$-expansionary and at which an increase is observed in the number of distinct available beginnings of $\Upsilon(\Xi\widehat{\Phi}^{A,B,D,A},A)$ which do not currently force $W^A$ —

- Or — see later — an increase of those beginnings $\Upsilon(\Xi\widehat{\Phi}^{A,B,D,A},A)|w$ — with $w$ an existing threshold for $\mathcal{R}$ for which there exists a $\sigma \supset \Upsilon(\Xi\widehat{\Phi}^{A,B,D,A},A)|w$ with $\sigma$ currently $\in W^A$, with the $A$-use of $W^A$ at $\sigma < \gamma(x), \lambda(\hat{x})$, but with $\sigma \not\in \Upsilon(\Xi\widehat{\Phi}^{A,B,D,A},A)$. 
(c) The activity of the \( \Upsilon \) module takes place at \( \Xi \hat{\Phi}^{A,B,D}_{A,A} \)-expansionary stages.

- In particular, new axioms for \( \Upsilon \) are enumerated at such stages.

- Subject to the qualifications appearing in phase \( 3^\Upsilon \) below — \( \Upsilon(\Xi \hat{\Phi}^{A,B,D}_{A,A}, A) \) is defined on arguments \( < \ell(\Xi \hat{\Phi}^{A,B,D}_{A,A}) \), with standard monotonic increasing use function \( \upsilon \).

(d) \( \Upsilon \) has use functions \( \upsilon^\Xi \) and \( \upsilon^A \) such that for each argument \( u \) of \( \Upsilon(\Xi \hat{\Phi}^{A,B,D}_{A,A}, A) \) one has

\[
(\upsilon^\Xi(u), \upsilon^A(u)) = \mu (a, b)[\Upsilon(\Xi \hat{\Phi}^{A,B,D}_{A,A} \upharpoonright a, A \upharpoonright b, u) ]
\]

and \( \upsilon^A(u) = \max \{ \xi(\upsilon^\Xi(u)), \varphi(\xi(\upsilon^\Xi(u))) \} \).

(e) The \( \Delta \) module for \( \tilde{\mathcal{P}} \) below \( \mathcal{R} \) acts at \( \mathcal{R} \)-expansionary stages.

- In particular — all new axioms for \( \Delta \) are enumerated at such stages — subject to the conditions in \( 3^\Delta \) below.
(f) $\Delta$ is assumed to have two distinct use functions $\delta^\Phi$ and $\delta^A$ such that for each argument $u$ of $\Delta \Omega(\hat{\Phi}^{A,B,D}, A)$ one has

$$ (\delta^\Phi(u), \delta^A(u)) $$

$$ = \mu(a, b)[\Delta(\Omega(\hat{\Phi}^{A,B,D} \uparrow a, A \uparrow a), A \uparrow b, u) \downarrow] $$

and

$$ \delta^A(u) = \hat{\varphi}(\delta^\Phi(u)). $$

(g) Assume that at stages at which $\lambda(\hat{x}) \downarrow$ any new $\gamma(x)$ is chosen $> \lambda(\hat{x})$.

- Write $\hat{\varphi}^\land \lambda(\hat{x}) = \text{the greatest } \hat{\varphi}(v) < \lambda(\hat{x})$, and $\hat{\Phi}^{A,B,D} \uparrow (\hat{\varphi}^\land \lambda(\hat{x}), \psi(y)]$ for $\hat{\Phi}^{A,B,D}$ restricted to arguments between $\hat{\varphi}^\land \lambda(\hat{x}) + 1$ and $\psi(y)$ —

- One assumes standard background activity to satisfy $\Omega(\hat{\Phi}^{A,B,D}, A) \neq \hat{\Theta}^A$, which will rely on the $\hat{\Phi}^{A,B,D}, A)$-changes arising from the failure of the $\Gamma/\Lambda$-strategy, along with relative honesty of the $\Omega$ uses.
The phases of the module:

1$\gamma$ / 1$\Delta$. Select a new $\tilde{y}$ as $D$-agitator for $\tilde{\mathcal{P}}$ (at $Q$ and $Q'$), with $\tilde{y} < \hat{y}$.

2$\gamma$ / 2$\Delta$. Select a new $\tilde{x} > \hat{x}$ to follow $\tilde{\mathcal{P}}$, and agitate via $\tilde{y} \setminus D$.

3$\gamma$. Select a new $w$ as a threshold for $\mathcal{R}$ at $\hat{\Phi}_{A,B,D}^A \uparrow (\hat{\phi} \lor \lambda(\hat{x}), \psi(y))$.

Register the following rules on the background activity:

- No subsequent new argument $v$ for $\Upsilon(\Xi \hat{\Phi}_{A,B,D}^A, A, A)$ is allowed to be defined at a stage at which $\Xi \hat{\Phi}_{A,B,D}^A \uparrow w$ does not exist.
- All new uses for $\Upsilon$ defined at later stages are $> w$. 
$3^\Delta$ (Simultaneous with $3^\gamma$).

Choose $\hat\Phi_{A,B,D} \upharpoonright (\varphi \lor \lambda(x), \psi(y))$, new for the $\Delta$ strategy, to follow $\mathcal{R}$ — while noting that no $\hat\Phi_{A,B,D} \upharpoonright (\varphi \lor \lambda(x), \psi(y))$ threatens to injure a computation $\Xi\hat\Phi_{A,B,D},A(v) = \Delta \Omega(\hat\Phi_{A,B,D},A),A(v)$, any $v \geq w$.

Register the following restriction on the background activity:

- Subsequent arguments for $\Delta \Omega(\hat\Phi_{A,B,D},A),A$ satisfy $\Delta \Omega(\hat\Phi_{A,B,D},A),A(v) = \Xi\hat\Phi_{A,B,D},A(v)$,
  $\delta\hat\Phi(v) = \delta^A(v) > \max\{\psi(y), \xi(v), \varphi(\xi(v))\}$ (that is, $\Delta$ is defined to be *honest* at $v$), — and with $\lambda(\hat{x})$ and $\gamma(x) > \hat{\varphi}(\xi(v))$ and $\hat{\varphi}(\delta\hat\Phi(v))$ (that is, $\Delta$ *avoids* $\Gamma/\Lambda$ at $v$), — where no such new argument $v$ is allowed unless it is accompanied by some $\sigma \supset \Upsilon(\Xi\hat\Phi_{A,B,D},A) \upharpoonright w$ with $\sigma \in W^A$. 
4\(\gamma\) / 4\(\Delta\). Wait for a \(\tilde{\mathcal{P}}\)-expansionary stage.

5\(\gamma\) / 5\(\Delta\). Ask: Is \(\tilde{\vartheta}(\tilde{x}) < \gamma(x), \lambda(\tilde{x})\)?

(a) Yes —

- Define \(B(\tilde{x}) \neq \tilde{\Theta}^A(\tilde{x})\).
- And restrain \(A \upharpoonright \tilde{\vartheta}(\tilde{x})\) and

  \[A, B, D \upharpoonright \text{max}\ \{\varphi'(\psi'(\tilde{y})), \hat{\varphi}'(\psi'(\tilde{y})), \psi'(\tilde{y})\}\].

(b) No —

- Activate 6\(\gamma\) — and return to 4\(\gamma\) / 4\(\Delta\) for \(\tilde{x}\).

6\(\gamma\). Ask: Is \(\Upsilon(\Xi\hat{\Phi}^{A,B,D,A}, A) \upharpoonright v \downarrow\) — and is it incompatible with its value at the antecedent implementation of 5\(\gamma\) / 5\(\Delta\), for some \(v < w\)?

I. No —
• Then ask: Does there exist a string $\sigma \supset \Upsilon(\Xi \hat{\Phi}^{A,B,D}_{A}, A) \upharpoonright w$ with $\sigma \in W^{A}$ — and with $\Xi \hat{\Phi}^{A,B,D}_{A} \upharpoonright w$ new for $\Upsilon$?

Case 1: Yes — Define $\sigma \subset \Upsilon(\Xi \hat{\Phi}^{A,B,D}_{A}, A)$.

• And appropriately restrain $A$, $B$ and $D$ to maintain this, and $\sigma \in W^{A}$, at later stages.

• Any $\Delta$ or $\overline{\Upsilon}$ strategy dependent on the failure of $\Upsilon$ is initialised.

Outcome: One maintains the $\Upsilon$ strategy, and $R$ is satisfied outright.

Case 2: No — Progress the $\Delta$ strategy for $R$ via $6^\Delta$ — and return to $4^\Upsilon / 4^\Delta$ for $\tilde{x}$.

II. No: Progress the auxiliary $\overline{\Upsilon}$ strategy for $R$ — and return to $4^\Upsilon / 4^\Delta$ for $\tilde{x}$.

$6^\Delta$. Let $y$, $\hat{y}$, $\check{y} \not\in D$, rectify $\Delta$ — and restrain $A$, $B$, $D \upharpoonright \max \{\varphi(\psi(\check{y})), \hat{\varphi}(\psi(\check{y})), \psi(\check{y})\}$ at $\tilde{P}$. 
Analysis of outcomes.

Impact on the level 1 and 2 outcomes:

- On $w_1$, $w'_1$, $w_2$ and $\hat{w}_2$ — None.

- On $s_1$, $\hat{s}_1$ — A similar argument to that for $s_1$ in the context of the level 2 module applies.

- On the infinitary outcomes $i_1$, $i'_1$ and $\hat{i}_1$ — These are now subsumed in the outcomes for the $\Upsilon$ strategy for $Q$.

Notice: No phase of the level 3 module involves any impediment to the eventual success of the $\Gamma/\Lambda$ strategy — or of the $\Gamma'/\Lambda'$ strategy — in the case of eventually incomplete fulfilment of the conditions for a successful $\Upsilon$ strategy.
Outcomes specific to the level 3 strategies:

\[ \tilde{w}_2 \]: $\tilde{\mathcal{P}}$ halts at $4^\gamma / 4^\Delta$.

- If there are only finitely many $\tilde{\mathcal{P}}$-expansionary stages, $\tilde{\mathcal{P}}$ is trivially satisfied.

- Otherwise — each progression of the $\Lambda$ strategy via $6^\Gamma$ involves $\gamma(x) \downarrow A$ — and each progression of the $\Upsilon$ strategy via $6^\Lambda$ involves $\lambda(\tilde{x}) \downarrow A$.

- And hence — if at every $\tilde{\mathcal{P}}$-expansionary stage one has $\gamma(x)$ or $\lambda(\tilde{x}) \leq \tilde{\vartheta}(\tilde{x})$ — one must eventually find $\tilde{\Theta}(\tilde{x}) \uparrow$.

\[ \tilde{w}_3 \]: Only finitely many $\Xi^{\tilde{\Phi}^{A,B,D}, A}$-expansionary stages.

- Then — in the context of an appropriate valid $\Upsilon$ strategy being passed down to lower priority requirements involving all such functionals $\Xi$ — $\mathcal{R}$ (and hence $\mathcal{Q}$) is satisfied.
\[ \tilde{w}_4 \]: Only finitely many \( \mathcal{R} \)-expansionary stages.

- Then — arguing as in the case of \( \tilde{w}_2 \) — will have some sufficiently long beginning \( \Upsilon(\Xi \hat{\Phi}^{A,B,D,A}_A, A) \upharpoonright w \) of \( \Upsilon(\Xi \hat{\Phi}^{A,B,D,A}_A, A) \) for which each \( \sigma \supset \Upsilon(\Xi \hat{\Phi}^{A,B,D,A}_A, A) \upharpoonright w \) with \( \sigma \in W^A \) — during a progression of the \( \Upsilon \) strategy — involves an \( A \)-use of \( W^A \) at \( \sigma \) which is \( \geq \gamma(x) \) or \( \lambda(\hat{x}) \) —

- Giving \( \mathcal{R} \) is satisfied via

\[
\Upsilon(\Xi \hat{\Phi}^{A,B,D,A}_A, A) \upharpoonright w \models W^A
\]

in the limit.

\[ \tilde{s}_1 \]: Phase 5\( ^{\Upsilon} / 5^{\Delta} \) applies.

- \( \tilde{\mathcal{P}} \) is satisfied —

- This satisfaction being eventually within the \( \Upsilon \) or \( \Delta \) strategy according as outcome \( \tilde{s}_2 \) or \( \tilde{s}_3 \) applies.
\[ \tilde{s}_2 \]: Phase 6\( ^\gamma \), part I, subcase 1 applies.

• In this case, \( Q \) is satisfied via the satisfaction of \( R \) due to \( \Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A) \models W^A \).

• One needs to verify:

  (a) Following 6\( ^\gamma \) — an intact strategy for building \( \Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A) \) below \( \Xi \hat{\Phi}^{A,B,D},A \oplus A \) exists.

  (b) \( \sigma \subset \Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A) \) at all sufficiently large stages. And:

  (c) \( \sigma \in W^A \) at all such stages.

• For (a) — The existence of infinitely many \( \Xi \hat{\Phi}^{A,B,D},A \)-expansionary stages ensure that there are beginnings of \( \Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A) \) of unbounded length defined —

• And — by the conditions of 6\( ^\gamma \), part I — any definition of \( \sigma \subset \Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A) \) via I, case 1, is consistent with existing axioms for \( \Upsilon \).
• For (b) — One observes that following such a definition of $\sigma \subset \Upsilon(\Xi_\hat{\Phi}^{A, B, D, A, A})$ — the accompanying restraints can only be injured by some existing parameter subsequently entering $A$, $B$ or $D$.

• But a current cycle of the $\Gamma$ strategy completed via $5^\Gamma$ involves no subsequent $A$-, $B$- or $D$-change capable of such injury —

• And any later cycle involves parameters new at the relevant application of $6^\Gamma$.

•• And (c) follows similarly.

\[
\bar{s}_3: \text{Phase } 6^\Delta \text{ applies.}
\]

•• In this case, the prospect of an eventual satisfaction of $\mathcal{R}$ via a successful $\Delta$-strategy is retained —

• While the potentiality for the $\Upsilon$ strategy being progressed via a subsequent outcome $\bar{s}_2$ for some other $\mathcal{P}$-requirement below $\hat{\mathcal{P}}$ below $\mathcal{Q}$ is maintained.
One needs to show that:

(a) Following $6^\Delta$ — an intact strategy for building $\Upsilon(\Xi \hat{\Phi}^{A,B,D},A,A)$ below $\Xi \hat{\Phi}^{A,B,D},A \oplus A$ exists (although outcome $\tilde{s}_2$ will not now be provided by $\tilde{\mathcal{P}}$).

(b) Following each application of $6^\Delta$, there is an intact strategy for building $\Delta \Omega(\hat{\Phi}^{A,B,D},A,A) = \Xi \hat{\Phi}^{A,B,D},A$, while

(c) For each $\nu — \Delta \Omega(\hat{\Phi}^{A,B,D},A,A)(\nu) \downarrow$ at all sufficiently large stages.

•• Clause (a) is immediate — since $6^\Delta$ only entails background enumeration of axioms for $\Upsilon$.

•• For (b) — assume $\Delta \Omega(\hat{\Phi}^{A,B,D},A,A)(\nu) \downarrow$ at some stage $t$ at which $\Xi \hat{\Phi}^{A,B,D},A(\nu) \uparrow$, or at which $\Xi \hat{\Phi}^{A,B,D},A(\nu) \downarrow \neq \Delta \Omega(\hat{\Phi}^{A,B,D},A,A)(\nu)$ —
• Hence $\Delta^{\Omega(\hat{\Phi}^{A,B,D},A),A}(v)$ gets defined equal to $\Xi^{\hat{\Phi}^{A,B,D},A}(v)$ at an $R$-expansionary stage $s$, say —

• And — at some later stage $t' \leq t$ — there occurs a relevant $A$, $B$, or $D$-change below $\max \{\xi(v), \varphi(\xi(v))\}$ or below $\hat{\varphi}(\delta^{\hat{\Phi}}(v))$ —

• Where — if the relevant change is an $A$-change — it is above $\delta^A(v)$.

• Since outcomes $s_1, \bar{s}_1, \tilde{s}_1$ and $\bar{s}_2$ do not apply —

• One can assume that such a change occurs via an application of phase $6^{\Gamma'} / 6^\Gamma$, $6^A$ or $5^\Gamma / 5^\Delta$ and then $6^\Delta$ — or via a progression of the $\bar{\Upsilon}$ strategy via $6^\Upsilon$, part II.

• Phases $6^{\Gamma'} / 6^\Gamma$ present no problems so long as $\Delta$ remains honest at $v$ —

• Since in that case any $\Xi^{\hat{\Phi}^{A,B,D},A}(v)$ change consequent on an $A\upharpoonright \varphi(\xi(v))$ or $A\upharpoonright \xi(v)$ change leads to eventual $\Delta$ rectification at $v$, without further $\hat{\Phi}^{A,B,D} \oplus A$ change — by the choice of $\delta^A(v)$ in $3^\Delta$.
• And any such change giving rise to a change in $\hat{\Phi}^{A,B,D} \uparrow \delta \hat{\Phi}(v)$ always leads to $\Delta$ rectification via an accompanying $A \uparrow \delta^A(v)$-change — so long as this can eventually be achieved with $\Delta$ avoiding $\Gamma/\Lambda$ at $v$, which will happen unless outcome $\bar{i}_2$ below applies.

• Similarly, $\lambda(x) \setminus A$ in $6^\Lambda$ is followed by rectification of $\Delta$ in $6^\Delta$ at all arguments of $\Delta \hat{\Phi}^{A,B,D,A}$ at which $\Delta$ is honest — again by the choice of $\delta^A(v)$.

• While — on the other hand — $\hat{\Phi}^{A,B,D} \uparrow \delta \hat{\Phi}(v)$ changes due to $A$ must again lead to appropriate $\Delta$ rectification at any relevant $v$.

• One also notices — by the above — that honesty of $\Delta$ at an argument $v$ cannot be lost via an implementation of $6^{\Gamma'}/6^\Gamma$ or $6^\Lambda$ alone.

• Say at a stage $< t'$ it happens that $\Delta$ becomes dishonest at an argument $v$ of $\Delta \hat{\Phi}^{A,B,D,A}$ via an application of $5^\tau/5^\Delta$ or $6^\Delta$ —
• Firstly — assume that \( \tilde{x} \searrow B \) via \( 5^\gamma / 5^\Delta \) at a stage \( t'' \), with \( s < t'' < t' \) and \( \tilde{x} \leq \varphi(\xi(v)) \) — resulting in \( \Delta \) becoming dishonest at \( v \) at a stage \( \leq t' \).

•• There are two cases:

  (i) \( s < \) the last previous stage at which \( 3^\Delta \) was implemented.

• But since \( \tilde{x} \) was chosen new via \( 2^\gamma / 2^\Delta \) — one has \( \tilde{x} > \varphi(\xi(v)) \) for all \( v \) for which \( \Delta \hat{\Phi}^{A,B,D,A}(v) \downarrow \) prior to the stage (\( s' \) say) at which \( 3^\Delta \) is implemented.

(ii) Otherwise.

•• In this case — one can easily check that the implementation of \( 5^\gamma / 5^\Delta \) is followed by that of \( 6^\Delta \) — leading to \( y, \hat{y} \) and \( \tilde{y} \nearrow D \), and a consequent return to the rectifying value of \( \hat{\Phi}^{A,B,D} \uparrow \psi(y) \) identified originally via \( 3^\Gamma \) —
• One only needs to verify that at no previous stage did one have $\Delta(\hat{\Phi}^{A,B,D}, A, v) \downarrow$ — with $A\upharpoonright \delta^A(v) \subseteq A[s']$ and $\hat{\Phi}^{A,B,D} \upharpoonright \delta^{\hat{\Phi}}(v)$ compatible with the rectifying value of $\hat{\Phi}^{A,B,D} \upharpoonright \psi(\tilde{y})$ — to ensure that in subsequently rectifying $\Delta$ one maintains honesty at $v$.

• But the restriction on background activity registered in any application of $3^\Delta$ ensures that $\delta^{\hat{\Phi}}(v) > \psi(y)$.

• And so if $\hat{\Phi}^{A,B,D} \upharpoonright \delta^{\hat{\Phi}}(v)$ is compatible with the rectifying value of $\hat{\Phi}^{A,B,D} \upharpoonright \psi(\tilde{y})$ at some such stage $> s'$ —

• One must have at that stage $\Phi^{A,B,D} \upharpoonright \psi(\tilde{y})$ being incompatible with the rectifying value of $\Phi^{A,B,D} \upharpoonright \psi(y)$ —

• Precluding a progression of the $\Delta$ strategy, and — in particular — any definition of new axioms for $\Delta$ at that stage.
• It now follows immediately that no such stage $t'$ exists at which one defines $B(\tilde{x}) \neq \tilde{\Theta}^A(\tilde{x})$ via phase $5^{\tilde{\gamma}} / 5^\Delta$ — or at which one implements $y, \hat{y}$ and $\hat{y} \not\in D$ via phase $6^\Delta$.

• And no progression of the $\overline{\Upsilon}$ strategy will materially alter the above argument — as will be seen from the analysis of the level 4 outcomes, below.

•• For (c) — the existence of infinitely many $\mathcal{R}$-expansionary stages ensure that unbounded beginnings of $\Delta^{\tilde{\Phi}^A,B,D,A}$ are defined.

• And hence the totality of $\Delta$ on argument $(\Omega(\tilde{\Phi}^{A,B,D,A}, A), A)$ easily follows from the above examination of $\Delta$-rectification in (b).

\[ \tilde{i}_2 \] : For some number $\nu$, $\Delta$ fails to avoid $\Gamma/\Lambda$ at $\nu$ at infinitely many stages.

•• In this case the infinitary progression of the $\Delta$ strategy accompanies infinitely many changes of $\lambda(\hat{x})$ and $\gamma(x)$ — leading to either $\Phi^{A,B,D}$ or $\Xi^E$ or $\tilde{\Phi}^{A,B,D}$ failing to be total.
Level 4: Module for $\tilde{\mathcal{P}}$ below $S'$ below $\tilde{\mathcal{P}}$ below $\mathcal{P}$, all acting below $Q$ below $Q'$ (with $\tilde{\mathcal{P}}$ successfully implementing the $\overline{\Upsilon}$ strategy for $Q$):

- Assume now that $\tilde{\mathcal{P}}$ implements phase $6^\Upsilon$, part II, infinitely often.
- The complete $\overline{\Upsilon}$ strategy will be provided by variant copies of $Q$ — with $\overline{W}^A$ in $S$ ranging over all $A$-c.e. sets — and $\overline{Q}$ unchanged from $Q$ outside of $S$.
- Let $S'$ be a typical $\mathcal{S}$-subrequirement of the form
  \[ \overline{\Upsilon}(\Xi^{\Phi^A,B,D,A},A) \vdash \overline{W}'^A \]
  activated above $\tilde{\mathcal{P}}$ via the level 3 module.
The $\overline{\Upsilon}$ strategy for $\mathcal{R}$ applied at $S'$:

Notes:

(a) As for the $\Upsilon$ strategy, this auxiliary strategy takes place at $\Xi \hat{\Phi}^{A,B,D,A}$-expansionary stages.

(b) The $S'$-expansionary stages are those $\mathcal{R}$-expansionary stages at which one also has an increase in the observed length of beginnings of $\overline{\Upsilon}(\Xi \hat{\Phi}^{A,B,D,A},A)$ not currently forcing $\overline{W}'^A$.

(c) Any new axioms for $\overline{\Upsilon}$ are enumerated on completion of a cycle of the module which concludes with an occurrence of part II of phase $6^\mathcal{R}$.

•• In working below $S'$ within the context of the $\overline{\Upsilon}$ strategy for $\mathcal{R} — \tilde{\mathcal{P}}$ requires the activity of the $\Upsilon$ strategy relative to $\tilde{\mathcal{P}}$ to synchronise with that of the $\overline{\Upsilon}$ strategy.
The phases of the module:

3⁻¹. Given a follower $\Phi^{A,B,D}_{} (\phi \lor \gamma(x), \psi(y))$ for $R$ — with corresponding threshold $w$ — Select a new $w' > w$ as a threshold for $R$ at $\Phi^{A,B,D}_{} (\phi \lor \gamma(x), \psi(y))$.

Register the following rules on the background activity:

- No subsequent new argument $v$ for $\Gamma(\Xi\Phi^{A,B,D}_{} , A)$ is allowed to be defined at a stage at which $\Xi\Phi^{A,B,D}_{} , A \uparrow w'$ does not exist.
- All new uses for $\Gamma$ defined at later stages are $> w'$.

Note: No new argument of $\Gamma(\Xi\Phi^{A,B,D}_{} , A)$ is allowed to have been defined since the previous visit to 6⁻¹, part II — so that $\Phi^{A,B,D}_{} (\phi \lor \gamma(x), \psi(y))$ threatens to injure no $\Xi\Phi^{A,B,D}_{} , A (v)$ used by an existing axiom for $\Gamma$. 
4. Wait for an \( S' \)-expansionary stages, with an accompanying \( \sigma' \supset \Upsilon(\Xi_{\Phi^{A,B,D},A}, A) \upharpoonright w' \) with \( \sigma' \in \overline{W'}^A \), with the use of \( \overline{W'}^A \) at \( \sigma' < \gamma(x), \lambda(\hat{x}) \).

5. Activate \( 5^\Upsilon / 5^\Delta \).

6. (Simultaneous with 6\( ^\Upsilon \), part II.) — Define \( \sigma' \subset \Upsilon(\Xi_{\Phi^{A,B,D},A}, A) \). Appropriately restrain \( A, B \) and \( D \) to maintain this and \( \sigma' \in \overline{W'}^A \) at later stages.

**Outcome:** One maintains the \( \Upsilon/\Delta \) strategy, while \( S' \) is satisfied.
Analysis of outcomes.

Impact on the higher level outcomes:

- On \( w_1, w'_1, w_2, \hat{w}_2, \tilde{w}_2, \tilde{w}_3 \) and \( \tilde{w}_4 \) — None.

- On \( s_1, \hat{s}_1, \tilde{s}_1, \tilde{s}_2 \) and \( \tilde{s}_3 \) — None.

- On the infinitary outcomes \( i_1, i'_1, \hat{i}_1 \) and \( \tilde{i}_2 \) — None.
Outcomes specific to the level 4 strategy:

$\tilde{w}_4$ : Only finitely many $S'$-expansionary stages.

- Then — as in the case of $\tilde{w}_4$ —

- There will be a sufficiently long beginning $\overline{\Upsilon}(\Xi^{\hat{\Phi}_{A,B,D}^A,A}, A) \upharpoonright w'$ of $\overline{\Upsilon}(\Xi^{\hat{\Phi}_{A,B,D}^A,A}, A)$ for which each $\sigma' \supset \overline{\Upsilon}(\Xi^{\hat{\Phi}_{A,B,D}^A,A}, A) \upharpoonright w'$, with $\sigma' \in \overline{W}'^A$, involves an $A$-use of $\overline{W}'^A$ at $\sigma'$ which is $\geq \gamma(x)$ or $\lambda(\hat{x})$ —

- Giving $S'$ satisfied via

  $\overline{\Upsilon}(\Xi^{\hat{\Phi}_{A,B,D}^A,A}, A) \upharpoonright w' \models \overline{W}'^A$.

$\tilde{s}_2$ : Phase 6 $\overline{\Upsilon}$, applies.

- Then $S'$ is satisfied due to

  $\overline{\Upsilon}(\Xi^{\hat{\Phi}_{A,B,D}^A,A}, A) \vdash \overline{W}'^A$

  — by a similar argument to that above for $\tilde{s}_2$.

One needs to verify —
(a) Following $6\overline{\Upsilon}$, there remains an intact strategy for building $\overline{\Upsilon}(\Xi\hat{\Phi}^{A,B,D},A,A)$ below $\Xi\hat{\Phi}^{A,B,D},A \oplus A$.

(b) $\sigma' \subset \overline{\Upsilon}(\Xi\hat{\Phi}^{A,B,D},A,A)$ at all sufficiently large stages. And —

(c) $\sigma' \in \overline{W}^{A}$ at all such stages.

••  For (a) — just notice that infinitely many $\Xi\hat{\Phi}^{A,B,D},A$-expansionary stages and occurrences of $6\overline{\Upsilon}$, part II, ensure that there are beginnings of $\overline{\Upsilon}(\Xi\hat{\Phi}^{A,B,D},A,A)$ of unbounded length defined.

•  The conditions of $6\overline{\Upsilon}$, part II — and the rules on the background activity registered in $3\overline{\Upsilon}$ — ensure that the definition of $\sigma' \subset \overline{\Upsilon}(\Xi\hat{\Phi}^{A,B,D},A,A)$ via $6\overline{\Upsilon}$ is consistent with existing axioms for $\overline{\Upsilon}$.

••  The arguments for (b) and (c) are almost the same as before.