

MATH 3163 COMPUTABILITY AND UNSOLVABILITY
Problems 7

1) Write an essay, covering *not more than three pages*, describing the background to, and consequences of, Alan Turing's discovery of the existence of a Universal Turing Machine.

Your answer should contain enough mathematical content to show a good grasp of the notions and results involved, and enough discussion of these to show an understanding of the broader context.

Some topics, a selection of which might be touched on in such an essay:

- (1) The importance of algorithms in the history of mathematics, and the way science focuses on identifying algorithmic content in the material universe.
- (2) The programmes of Leibniz, Hilbert for algorithmically capturing logic and mathematics.
- (3) The various formulations of the notion of a computable function, and their importance to an examination of the feasibility of Hilbert's programme.
- (4) The notion of a Turing machine and how a TM computes, and how to obtain a Universal Turing Machine U .
- (5) How such a U can be used to reveal the existence of incomputable sets and unsolvable problems. Links with logic.
- (6) Computationally enumerable sets. Incomputability and the arithmetical hierarchy (the connection with Σ_1^0 sets).
- (7) How U anticipates the stored program computer.
- (8) The Church-Turing Thesis ...
- (9) ... and its extensions to the real world — evidence either way?
- (10) The human mind as a Turing machine?
- (11) Hilbert's 10th Problem, and the search for diverse natural examples of incomputable sets.
- (12) Oracle TMs, and the resulting structure (the Turing universe).
- (13) The Turing universe as a model for computably complex environments.

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