University of Leeds
Department of Pure Mathematics

MATH 5164M ADVANCED COMPUTABILITY AND UNSOLVABILITY
Problems 7

1) "There is nothing *useful* one can say about incomputability" — discuss.
Write an essay answering the above question, covering *not more than three pages*.
Your answer should contain enough mathematical content to show a good grasp of the notions and results involved in analysing incomputable sets, relations and functions, and enough discussion of these to show an understanding of the broader context.

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Some topics, a selection of which might be touched on in such an essay:

- The discovery of various models of computability, and the way they express our intuitive notion of what it means for a mathematical object to be computable.
- Using such models to reveal the existence of incomputable sets and unsolvable problems.
- The relationship of such unsolvable problems to the programmes of Leibniz and Hilbert for algorithmically capturing logic and mathematics.
- Natural examples of incomputable sets, and the notion of a computably enumerable set. Relationship to emergence and computer simulations.
- How the analysis of incomputability can throw light on what we can and cannot compute, and point to new paradigms of computation which may go beyond computers based on the Turing machine model.
- The elusiveness of natural examples of incomputable sets in nature, and the extent to which everyday mathematics is captured in standard axiomatic systems.
- Circumstantial evidence for incomputability in nature and everyday mathematics.
- Oracle TMs, and the resulting structure (the Turing universe).
- Ways of analysing incomputability, and of providing models for environments exhibiting incomputability — e.g., degree structures (Turing and many-one degrees), hierarchies (arithmetical hierarchy).
- The Church-Turing Thesis extended to the physical universe.
- The human mind as a Turing machine?
- Incomputability, chaos, and emergence.
- Incomputability and quantum phenomena. Quantum computation.
- Incomputability and relativistic phenomena, e.g., black holes.

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