1) Show that $\equiv_T$ is an equivalence relation.

2) Show that $\leq$ is a partial ordering on $\mathcal{D}$.

3) (a) Show that for all $A, B \subseteq \mathbb{N}$
   i) $A \leq_T A \oplus B$ and $B \leq_T A \oplus B$, and
   ii) If $A \leq_T C$ and $B \leq_T C$ then $A \oplus B \leq_T C$.

   (b) If we define the join $a \cup b$ of Turing degrees $a = \deg(A), b = \deg(B)$ by $a \cup b = \deg(A \oplus B)$, deduce that $a \cup b = \text{lub}\{a, b\}$.

4) Show that there is a least Turing degree $0 = \text{the set of all computable sets}$.

5) Show that:
   (a) If $X \subseteq \mathbb{N}$ is $A$-computable then $X$ is $A$-c.e.
   (b) $X$ is $A$-computable if and only if $X$ and $\overline{X}$ are $A$-c.e. (if and only if $X \in \Delta^A_1$, where we write $X \in \Delta^A_1$ for $X \in \Sigma^A_1$ and $X \in \Pi^A_1$).
   (c) $X$ is $A$-c.e. if and only if $X \in \Sigma^A_1$.

6) Show that:
   (a) $X \leq_m A'$ if and only if $X$ is $A$-c.e.
   (b) If $K^A = \{x \mid x \in W^A_x\}$, then $K^A$ is $A$-c.e. but not $A$-computable.

7) Show that there exists an infinite sequence $a_0, a_1, \ldots$ of degrees $\leq 0'$ such that for each $i \neq j$ we have $a_i \mid a_j$.

For MATH5164M only:

8) For any string $\sigma$ let $\bar{\sigma}$ be given by

   $$\bar{\sigma} = \begin{cases} 
   0 & \text{if } \sigma(x) = 1, \\
   1 & \text{if } \sigma(x) = 0.
   \end{cases}$$

   Show that if $\hat{W}_i$ is a c.e. set of strings then so is $\{\bar{\sigma} \mid \sigma \in \hat{W}_i\}$.

   Hence, or otherwise, show that $A \subseteq \mathbb{N}$ is $1$-generic if and only if $\overline{A}$ is $1$-generic.

9) We say that $A \subseteq \mathbb{N}$ is immune if and only if $A$ is infinite and contains no infinite c.e. subsets.

   Show that for each $i$ $Y_i = \{\sigma \mid \exists x[\sigma(x) = 0 \& x \in W_i]\}$ is a c.e. set of strings.

   Show that if $A$ is $1$-generic then $A$ forces each such $Y_i$, and hence that $A$ is either finite or immune.

   Deduce (using the result of question 8 above) that every $1$-generic set is immune, and hence not c.e.