1) Show that the following functions are primitive recursive:

(i) \( \overline{sg}(n) = \begin{cases} 1 & \text{if } n = 0 \\ 0 & \text{otherwise} \end{cases} \),

(ii) \( n! = \text{factorial} n \),

(iii) \( \min(m, n) = \begin{cases} m & \text{if } m \leq n \\ n & \text{otherwise} \end{cases} \),

(iv) \( \min\{m_1, \ldots, m_n\} = \text{least of the numbers } m_1, \ldots, m_n \) (n given),

(v) \( \max\{m_1, \ldots, m_n\} = \text{largest of the numbers } m_1, \ldots, m_n \),

(vi) \( qt(m, n) = \text{quotient on division of } n \text{ by } m \).

2) Show that if \( R, S \) are primitive recursive sets, so are \( \mathbb{N} - R, R \cap S \) and \( R \cup S \).

3) Show that every finite set is primitive recursive.

4) Show that if \( R(n) \) is a recursive relation, and \( f \) is a recursive function, then \( R(f(n)) \) is a recursive relation.

5) Show that if \( f \) is a recursive function with infinite range, then we can find a 1-1 recursive function \( g \) with range \( f = \text{range } g \).

6) Let \( h(\vec{n}, m, p) \) be primitive recursive. Let \( f(\vec{n}, m + 1) = h(\vec{n}, m, \tilde{f}(\vec{n}, m)) \), where

\[
\tilde{f}(\vec{n}, m) \overset{\text{defn}}{=} p_0^{f(\vec{n}, 0)} \times \cdots \times p_m^{f(\vec{n}, m)},
\]

and \( f(\vec{n}, 0) \) is primitive recursive. Show that \( \tilde{f} \), and hence \( f \), is primitive recursive.

7) Let \( h_0(\vec{n}), \ldots, h_k(\vec{n}) \) be primitive recursive, and let \( R_0(\vec{n}), \ldots, R_k(\vec{n}) \) be primitive recursive relations, exactly one of which holds for any given \( \vec{n} \). Show that if

\[
f(\vec{n}) \overset{\text{defn}}{=} \begin{cases} h_0(\vec{n}) & \text{if } R_0(\vec{n}) \\ \vdots & \vdots \\ h_k(\vec{n}) & \text{if } R_k(\vec{n}) \end{cases},
\]

then \( f \) is primitive recursive.
8) A Fibonacci sequence \( \{u_n\}_{n \geq 0} \) is given by

\[
    u_0 = k_0, \quad u_1 = k_1, \quad u_{n+2} = u_{n+1} + u_n.
\]

Show that \( u_n \) is a primitive recursive function.

HAND IN A SOLUTION TO QUESTION 1 AND TWO OTHER QUESTIONS.