University of Leeds
Department of Pure Mathematics

MATH 3163/5164M – COMPUTABILITY & UNSOLVABILITY

Course Outline

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Recommended text: S. Barry Cooper, *Computability Theory*, Chapman & Hall/C.R.C., hardback, Dec. 2004.
Background reading: Roger Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Arrow Books, 1989, paperback.
Andrew Hodges, *Alan Turing: The Enigma*, Arrow Books, 1992, paperback.
Martin Davis, *The Universal Computer: The Road from Leibniz to Turing*, W.W. Norton, 2000.

§1. HISTORICAL BACKGROUND:

Hilbert, algorithms and formalism; Gödel, Church, Turing and the formalisation of computability – and the discovery of incomputability; the consequences for mathematics and the 'real world'. [See **CT**, pages 3–10.]

§2. RECURSIVE FUNCTIONS:

Primitive recursive functions; the μ-operator and (partial) recursive functions; Church's thesis; (primitive) recursive relations and sets. [**CT**, pages 11–25.]

§3. TURING MACHINES:

Turing programmes and Turing computable functions; the Church-Turing thesis. [**CT**, pages 34–44.]

§4. THE ENUMERATION THEOREM:

Gödel numbers for Turing machines, the e^{th} Turing machine, the Universal Turing Machine, and its theoretical anticipation of the stored programme computer. The Fixed Point Theorem. [**CT**, pages 61–68.]

§5. COMPUTABLY ENUMERABLE SETS AND UNSOLVABLE PROBLEMS:

Computably enumerable (c.e.) and computable sets; Σ^0_1 sets and the Normal Form Theorem for c.e. sets; an incomputable c.e. set, and Turing machines with unsolvable halting problems; creative and simple sets. [**CT**, pages 69–81 and 87–94.]

§6. HILBERT'S TENTH PROBLEM:

Diophantine equations and sets; Davis' strategy and Matiasevich's theorem on the diophantine nature of Fibonacci sequences. [**CT**, pages 94–99.]

§7. THE NONCOMPUTABLE UNIVERSE: I. MANY-ONE REDUCIBILITY:

Models of computationally complex environments; many-one reducibility and equivalence, and the the many-one degrees as a first attempt at a model. [**CT**, pages 101–115.]

§8. THE NONCOMPUTABLE UNIVERSE: II. TURING DEGREES:

Turing machines with oracles, and computability relative to auxiliary information; basic properties of the degrees of unsolvability (or Turing degrees) $\boldsymbol{\mathcal{D}}$; the jump operator, and the Jump Theorem. [**CT**, pages 139–154 and 161–166.]

§9. FORCING AND STRUCTURE:

Forcing in the context of computability theory; an existence theorem for 1-generic sets, and applications; low degrees and sets which force their jumps; the basic structure of $\boldsymbol{\mathcal{D}}$. [**CT**, pages 287–296.]

§10. PRIORITY CONSTRUCTIONS AND THE LOCAL THEORY:

The computably enumerable degrees; the Friedberg-Muchnik theorem and finite injury priority arguments; further applications and developments. [**CT**, pages 237–246.]

NOTE: There will be six sets of problems, and these will be an essential part of the course. The marks for the best *FIVE* sets of problems handed in will contribute 15% to the final grade (3 marks for each set of problems).
In the final examination, NO CALCULATORS WILL BE ALLOWED.