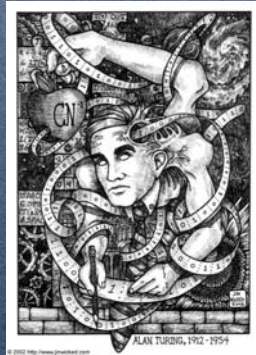


Computability and Unsolvability

- a brief chronology

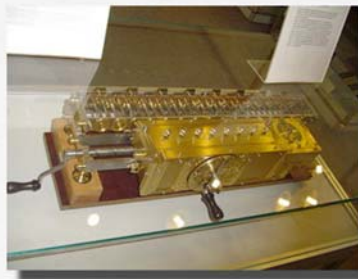


The Algorithmic Content of the Real World

- 1672-ish - Gottfried **LEIBNIZ**'s 'stepped reckoner' - He dreams of reduction of all reasoning to calculation



...it is beneath the dignity of excellent men to waste their time in calculation when any peasant could do the work just as accurately with the aid of a machine.



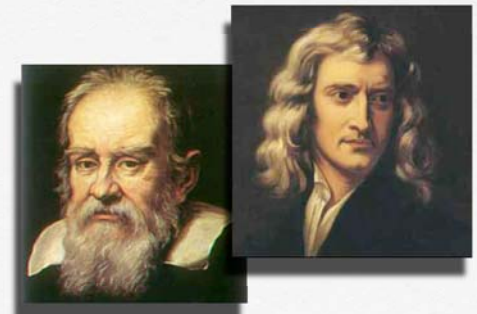
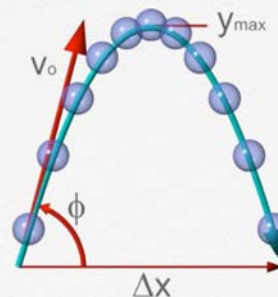
The Algorithmic Content of the Real World

- c.330 BC - **EUCLID**'s G.C.D. algorithm - earliest known
- 9th Century - **Muhammad ibn Musa AL-KHOWARIZMI** gives his name to the notion of 'algorithm'



Prediction & Real World Algorithmic Content

- Galileo and Newton onwards - overarching aim of **science** became the **extraction of the algorithmic content** of the world - a 'clockwork universe'





Laplace's Predictive 'Demon' as model

"Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situations of the beings who compose it - an intelligence sufficiently vast to submit these data to analysis - it would embrace in the same formula the movements of the greatest bodies and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes."

from P. S. de Laplace [1819], "Essai philosophique sur les probabilités"

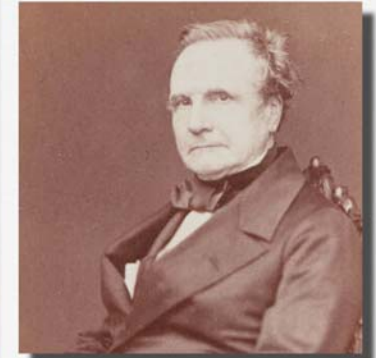
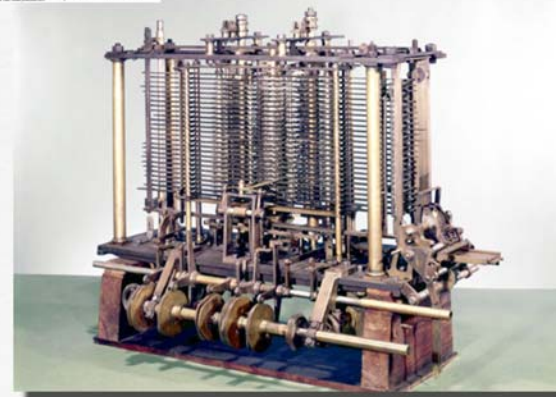


5



An Anticipation of the Computer

- 1834 - CHARLES BABBAGE and the 'Analytic Engine'

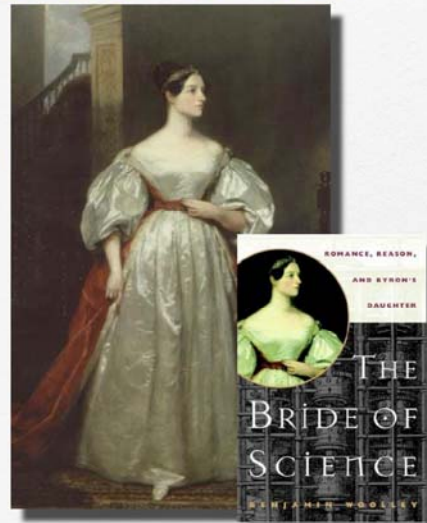


6



An Anticipation of the Computer

- 1815-1852 - ADA LOVELACE, daughter of Lord Byron, the "first programmer"
- Computing language Ada named after her by the US Defence Dept.
- Lovelace Medal of BCS set up



7



Hilbert's Programme

- c.1900 - David HILBERT and formalism

"For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either. ... The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem.

*In contrast to the foolish Ignorabimus, our credo avers:
We must know,
We shall know. "*

- David Hilbert's opening address to the Society of German Scientists and Physicians, Königsberg, September 1930



8



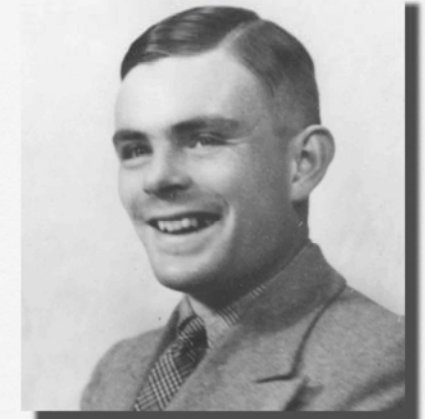
Gödel's Incompleteness Theorem



- 1931 onwards - GÖDEL'S INCOMPLETENESS THEOREM + various formalisations of the notion of an object being "effectively computable" -
- Recursive functions (Gödel, Kleene)
- λ -computable functions (Church, Kleene)
- Turing computable functions



The Discovery of Incomputable Objects



- 1936 - CHURCH, TURING - there is no computable test for logical validity
- The UNIVERSAL TURING MACHINE discovered - a machine with unsolvable Halting Problem



New Algorithmic Content ...

- Incomputable computably enumerable sets
- Approximations to Δ_2 and Σ_2 sets



"... if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."

A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted by Andrew Hodges in "Alan Turing - the enigma", p.361



What Came Later ...

- Late 1930s on - Development work on early computers + theory of computable functions (KLEENE, POST, etc)
- 1950s to present - Discovery of more and more basic examples of incomputable phenomena -
- Negative solution to Hilbert's Tenth Problem

CLAY MATHEMATICS INSTITUTE • MUSEUM OF SCIENCE

Julia Robinson and Hilbert's Tenth Problem

Screened from a film by George Colceary

Film Screening
Thursday, March 15 at 7:30pm
Cahners Theater
Museum of Science
Science Park, Boston

Followed by a panel discussion with George Colceary, Tom Matteson, and Catherine Stoll

George Colceary, Documentary Director
Tom Matteson, Director of the Museum of Science
Catherine Stoll, Director of the Cahners Theater
Screened in the context of Hilbert's Tenth Problem
A.M. Turing and other mathematicians

Julia Robinson (1919 - 2012) was the only woman to receive the Fields Medal, the highest honor in mathematics. She was a pioneer in the study of Hilbert's Tenth Problem, which asks whether there is an algorithm that can determine whether a given Diophantine equation has integer solutions.

OPEN TO THE PUBLIC

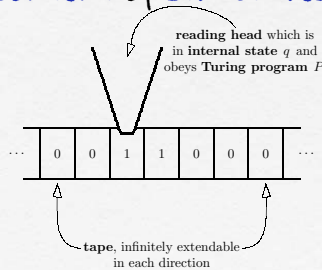
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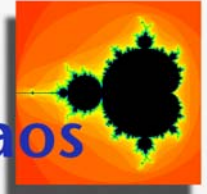


The Turing model extended

- 1939 - Turing's oracle Turing machines appear
- Provides a model of algorithmic content of structures which are based on the reals
- Modelling role of **ORACLE TURING MACHINES**, and of **TURING UNIVERSE**



Mathematical analogues of chaos



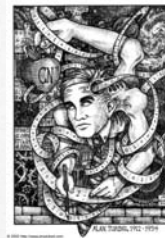
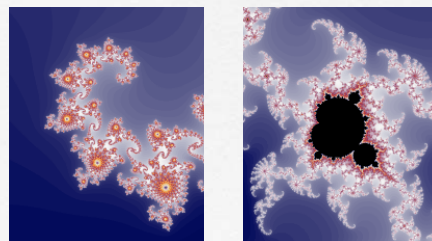
- Growth of **Chaos theory**, generation of informational complexity via very simple rules, accompanied by the emergence of new regularities - e.g. Robert Shaw's dripping tap [1984]
- Link** between structures in nature, and mathematical objects, such as the Mandelbrot and Julia sets
- Penrose, Smale - **computability** of Mandelbrot, Julia sets?



Now we witnessed ... a certain extraordinarily complicated looking set, namely the Mandelbrot set. Although the rules which provide its definition are surprisingly simple, the set itself exhibits an endless variety of highly elaborate structures.

Roger Penrose
in "The Emperor's New Mind", Oxford Univ. Press, 1994

Is the Mandelbrot set computable?



Emergence of patterns in Nature



1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.



From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:





Incomputability and the Human Mind

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press



Incomputability and the Human Mind

We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.



photo credit: www.harryjordan.com

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997