MATH3102 MATHEMATICAL LOGIC 2/MATH5103M ADVANCED LOGIC
Problems 1

1) Show that the following formulas are theorems of PA:

(PA2)$'$ \( t_1 = t_2 \rightarrow t'_1 = t'_2 \)
(PA3)$'$ \( 0 \neq (t_1)' \)

(where \( t_1, t_2 \) are terms of PA).

2) Show that \( t = r \vdash_{PA} (r = s \rightarrow t = s) \), and hence that \( \vdash_{PA} t = r \rightarrow (r = s \rightarrow t = s) \).

3)\( ^D \) Show that \( r = t, s = t \vdash_{PA} r = s \), and hence that \( \vdash_{PA} r = t \rightarrow (s = t \rightarrow r = s) \).

4)\( ^D \) Show by induction (in the metalanguage) that \( \vdash_{PA} \overline{m} + \overline{n} = \overline{m} + \overline{n} \) for any non-negative integers \( m \) and \( n \).

[In questions 2-4, you can assume the Deduction Theorem, \( \vdash_{PA} t = r \rightarrow r = t \), and all the theorems \( (PA1)' - (PA8)' \) got by replacing variables by terms in \( (PA1) - (PA8) \).]