

University of Leeds  
Department of Pure Mathematics

**MATH3102 – MATHEMATICAL LOGIC 2/ MATH5103M – ADVANCED LOGIC**  
**Course Outline**

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**Office hours:** Wednesdays 1.00–2.00pm.

**Recommended text:** S. Barry Cooper, *Computability Theory*, Chapman & Hall/C.R.C., hardback, Dec. 2004.

**Alternative reference:** A. G. Hamilton, *Logic for Mathematicians*, Cambridge University Press, revised edition, Sep. 1988, paperback).

**Background reading:** John Dawson, *Logical Dilemmas: Life and Work of Kurt Gödel*, A K Peters, paperback edition, 2005.

Douglas R Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, paperback edition, 2000.

Roger Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, Arrow Books, 1999, paperback.

**§1. HISTORICAL BACKGROUND:**

Cantor, Frege and the paradoxes. Solutions to the problem of paradoxes. Hilbert and formalism, Brouwer and intuitionism. Hilbert's Programme. Completeness, incompleteness, and Gödel's Theorem.

**§2. REVISION OF FIRST-ORDER LOGIC:**

The first-order language  $\mathcal{L}$ ; formulas and sentences; the axiomatisation  $\mathcal{K}$  for predicate calculus; proofs, deductions, theorems; the Deduction Theorem; Gödel's Completeness Theorem for  $\mathcal{K}$ ; Löwenheim-Skolem Theorem and the Compactness Theorem.

**§3. FIRST-ORDER PEANO ARITHMETIC:**

The language and special axioms of  $PA$ ; proofs in  $PA$ ; models of  $PA$ ; non-standard models.

**§4. REPRESENTABILITY IN  $PA$ :**

Examples of number-theoretic functions and relations; representability and functionally represents.

## §5. THE RECURSIVE FUNCTIONS:

Primitive recursive functions; the  $\mu$ -operator; partial recursive functions. Church's Thesis. Recursive relations.

## §6. REPRESENTABILITY OF RECURSIVE FUNCTIONS:

An outline proof that all recursive functions and relations are representable in  $PA$ . The representability of all computable functions, relations and sets.

## §7. ARITHMETISING ARITHMETIC:

Gödel numbers. Important number theoretic relations expressing aspects of  $PA$ , and their recursiveness. Recursive/computable axiomatisability.

## §8. COMPUTABLY ENUMERABLE SETS AND MANY-ONE REDUCIBILITY:

Computably enumerable (c.e.) and many-one reducibility; a computably axiomatisable theory has a c.e. set of theorems. Consistency and  $\omega$ -consistency. Semi-representability. A set is c.e. iff semi-representable in  $PA$ , iff many-one reducible to the set of theorems of  $PA$ . Enumeration theorem for c.e. sets. A c.e. set  $K$  which is not computable.

## §9. GÖDEL'S INCOMPLETENESS THEOREM:

Completeness of a theory; using representability of  $K$  to prove incompleteness of a theory; incompleteness of  $PA$ . Extensions of a theory. Gödel's First and Second Incompleteness Theorems.

## §10. UNDECIDABILITY AND CREATIVE SETS:

Decidability of a theory. Creative sets. The creativeness of  $PA$ .

## §11. CHURCH'S THEOREM:

Raphael Robinson's finite axiomatisation of first-order arithmetic. Church's Theorem and the undecidability of logical validity. The creativeness of  $\mathcal{K}$ .

## §12. MORE ON LOGIC AND COMPUTABILITY (for MATH 5103M):

We will look further at logic as a branch of computability theory, dealing with Turing machines and Turing's notion of oracle machine. Further topics from: Relative computability and degree structures; Computably enumerable sets; Arithmetical hierarchy and forcing in computability theory.

**NOTE: There will be six sets of problems. These will be an essential part of the course, but will not count towards the final grade. In the final examination, NO CALCULATORS WILL BE ALLOWED.**