

THE PRIMITIVE RECURSIVE FUNCTIONS:

1) The *initial functions* are primitive recursive:

(a) The *zero function* defined by

$$\mathbf{0}(n) = 0, \quad \forall n \in \mathbb{N},$$

(b) The *successor function* defined by

$$n' = n + 1, \quad \forall n \in \mathbb{N},$$

(c) The *projection functions* U_i^k defined by

$$U_i^k(\vec{m}) = m_i, \quad \text{each } i = 1, \dots, k,$$

(where $\vec{m} = m_1, \dots, m_k$).

2) If g, h, h_0, \dots, h_l are primitive recursive, then so is f obtained from g, h, h_0, \dots, h_l by one of the rules:

(d) *Substitution*, given by:

$$f(\vec{m}) = g(h_0(\vec{m}), \dots, h_l(\vec{m})),$$

(e) *Primitive recursion*, given by:

$$\begin{aligned} f(\vec{m}, 0) &= g(\vec{m}), \\ f(\vec{m}, n + 1) &= h(\vec{m}, n, f(\vec{m}, n)). \end{aligned}$$