

MATH 3032 GRAPH THEORY – TERMINOLOGY AND NOTATION

LIST 7

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**66)  $k$ -COLOURABLE,  $k$ -CHROMATIC,  $\chi(G)$**  —  $G$  is  **$k$ -colourable** if each of the vertices of  $G$  can be assigned one of  $k$  colours in such a way that no 2 adjacent vertices have the same colour.

$G$  is  **$k$ -chromatic** if  $G$  is  $k$ -colourable but not  $(k - 1)$ -colourable. The **chromatic number** of  $G$  is defined by:  $\chi(G) = k \Leftrightarrow G$  is  $k$ -chromatic.

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**67) CRITICAL,  $k$ -CRITICAL** —  $G$  is **critical** if  $\chi(H) < \chi(G)$  for every proper subgraph  $H$  of  $G$ .  $G$  is  **$k$ -critical** if  $\chi(G) = k$  and  $G$  is critical.

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**68)  $\delta$ ,  $\Delta$**  —  $\delta(G)$  = the smallest vertex degree in  $G$ , and  $\Delta(G)$  = the largest vertex degree in  $G$ .

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**69) MAP,  $k$ -FACE-COLOURABLE, FACE-CHROMATIC NUMBER  $\chi^*(G)$**  — A **map** is a plane graph with no cut edges.

A map is  **$k$ -face-colourable** if its faces can be coloured using  $k$  colours in such a way that no two faces having the same colour are adjacent.

The **face-chromatic number**  $\chi^*(G)$  for a map  $G$  is the least  $k$  for which  $G$  is  $k$ -face-colourable.

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**70)  $k$ -EDGE COLOURABLE, CHROMATIC INDEX  $\chi'(G)$**  — A graph is  **$k$ -edge colourable** if its edges can be coloured using  $k$  colours in such a way that no two adjacent edges have the same colour.

The **chromatic index**  $\chi'(G)$  for a loopless graph  $G$  is the least  $k$  for which  $G$  is  $k$ -edge colourable.

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**71) TAIT COLOURING** — A **Tait colouring** of a cubic graph  $G$  is a 3-edge colouring of  $G$  (in which no 2 adjacent edges take the same colour).

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**72) CHROMATIC NUMBER  $\chi(S)$  of a surface  $S$**  — The **chromatic number**  $\chi(S)$  of  $S$  is the number of colours needed to properly colour the faces of any map on  $S$ .

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**73) EULER CHARACTERISTIC** — The **Euler characteristic**  $n(S)$  of  $S$  is given by  $n = \nu - \varepsilon + \varphi$  (evaluated for a suitable graph  $G$  embedded on  $S$ ).

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**74) GENUS  $g(S)$**  — The **genus**  $g(S)$  of a surface  $S$  is the greatest number of distinct continuous non-selfintersecting closed curves which can be drawn on  $S$  without separating it into distinct regions.

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