50) **DUAL GRAPH** $G^*$ — The dual of $G$, written $G^*$, is got from the plane graph $G$ by the following rules:

(a) Corresponding to a face $f$ of $G$, we take a vertex $f^*$ of $G^*$,

(b) Corresponding to an edge $e$ of $G$, we take an edge $e^*$ of $G^*$,

(c) $f^*$, $g^*$ are joined by $e^*$ in $G^*$ $\iff$ $f, g$ are separated by $e$ in $G$.

51) **SUBDIVISION** — A subdivision of a graph $G$ is a graph that can be obtained from $G$ by subdividing edges of $G$ by adding extra vertices.

52) **DIRECTED GRAPH** or **DIGRAPH**, **VERTICES**, **ARCS**, **HEAD**, **TAIL**, **SUB-DIGRAPH** — A directed graph or digraph $D(V, A)$ is a set $V$ of vertices together with a set $A$ of ordered pairs $(u, v)$ ($u, v \in V$) called arcs.

$v$ is the **head** and $u$ the **tail** of $(u, v)$ and $u, v$ are **endpoints**. We say $(u, v)$ **joins** $u$ to $v$.

The digraph $D'$ is a **subdigraph** of $D$ if $V(D') \subseteq V(D)$ and $A(D') \subseteq A(D)$.

53) **UNDERLYING GRAPH**, **ORIENTATION** — From a given digraph $D$ we can obtain an undirected graph $G$ by letting $G$ have vertex set $V = V(D)$ and edge-set $E = \{ uv \mid (u, v) \text{ is an arc of } D \}$. $G$ is the **underlying graph** of $D$.

Conversely, $D$ is an **orientation** of $G$ if $G$ is the underlying graph of $D$.

54) **DIRECTED ARC SEQUENCE**, **DICHAIN**, **DIPATH**, **DICIRCUIT** — A directed arc sequence is a finite sequence of the form $(u_1, u_2), (u_2, u_3), \ldots, (u_{n-1}, u_n)$. This sequence is a **dichain** if all the arcs are distinct. The sequence is a **dipath** if all the vertices are distinct. The sequence is a **dicircuit** if it is a dipath with $u_1 = u_n$.

55) **INDEGREE**, **OUTDEGREE** — The **indegree** $d_D^-(u)$ of $u$ is the number of arcs of $D$ with head $u$. The **outdegree** $d_D^+(u)$ of $u$ is the number of arcs of $D$ with tail $u$.

56) $k$-**regular**, **regular** — $D$ is a $k$-**regular** digraph if $d^-(u) = d^+(u) = k$ for each vertex $u$ of $D$. $D$ is **regular** if $k$-regular for some $k$.

57) **REACHABLE**, **STRONGLY CONNECTED**, **STRONG COMPONENT** or **DICOMPONENT** — $v$ is **reachable** from $u$ if there is a dipath with initial vertex $u$ and terminal vertex $v$.

$D$ is **strongly connected** if, for each pair $u, v$ of vertices of $D$, $u$ is reachable from $v$ and $v$ is reachable from $u$.

A **strong component** or **diconnected** of $D$ is a strongly connected subgraph with the maximum possible number of arcs.