1) For which values of \( m \) and \( n \) are the following graphs Eulerian? (i) \( K_{m,n} \), (ii) \( K_n \).

2) Say (giving reasons) which of the following figures can be drawn without lifting one’s pen from the paper or covering a line more than once?

3) If possible, draw an Eulerian graph \( G \) with \( v \) even and \( \varepsilon \) odd; otherwise, explain why there is no such graph.

4) Show that the Peterson graph is non-Hamiltonian; is it semi-Hamiltonian?

5) Give an example of a graph which is Eulerian but not Hamiltonian, and one which is Hamiltonian but not Eulerian.

What can you say about graphs which are both Eulerian and Hamiltonian?

6) Is it possible for a knight to travel round an \( 8 \times 8 \) chessboard in such a way that every possible move occurs exactly once?

7) Is it possible for a knight to visit all the squares of an \( 8 \times 8 \) chessboard exactly once, and then return to its starting point?

How about the same problem with a \( 7 \times 7 \) board?

8) Show that if \( G \) has a Hamilton path then, for every proper subset \( S \) of \( V \),

\[
c(G - S) \leq |S| + 1.
\]

9) Give a counter-example to show that in the sufficient condition of Dirac for \( G \) to be Hamiltonian, we cannot replace “\( d(u) \geq \frac{v}{2} \)” by “\( d(u) \geq \frac{v}{2} - 1 \)”.
10) A mouse eats her way through a $3 \times 3 \times 3$ cube of cheese by tunnelling through all of the $27$ $1 \times 1 \times 1$ subcubes. If she starts at one corner and always moves on to an uneaten subcube, can she finish at the centre of the cube?

11) $G$ is **hypohamiltonian** if $G$ is not Hamiltonian but $G[V - \{v\}]$ is Hamiltonian for every $v \in V$.

Show that the Peterson graph is hypohamiltonian.

12) $G$ is **hypotraceable** if $G$ has no Hamilton path but $G - v$ has a Hamilton path for every $v \in V$.

Show that the *Thomassen graph* below is hypotraceable.

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Hand in **FOUR** questions, including at least **ONE** of the more difficult questions, marked “D”. 

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