University of Leeds
Department of Pure Mathematics

# MATH 2040 MATHEMATICAL LOGIC I

## Course Outline

**Lecturer:** Prof. S. Barry Cooper.
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**Office hours:** Thursdays 1.30–2.30pm.

**Some useful texts:** J. Barwise and J. Etchemendy, *The Language of First Order Logic*, CSLI Publications.

R. Cori and D. Lascar, *Mathematical Logic – Part I*, OUP, 2000.

D. van Dalen, *Logic and Structure*, Springer-Verlag, 1980.

J.K. Truss, *Discrete Mathematics for Computer Scientists*, Springer-Verlag, 1980.

**Background on logic and computability:** S. Barry Cooper, *Computability Theory*, Chapman & Hall/C.R.C., hardback, Dec. 2003.

### §1. HISTORICAL BACKGROUND:

Origins of logic in ancient China, India, Greece; Descartes and the beginnings of formal logic; Frege and symbolic logic; the paradoxes and Hilbert's Programme; Gödel, Turing and the incomputable.

### §2. PROPOSITIONAL LOGIC:

The symbols of propositional logic; reasoning with propositional symbols; the formulae of propositional logic; parsing and Polish notation; the distinction between *syntax* and *semantics*; interpretations.

### §3. INTERPRETATIONS:

Semantics: truth-tables; tautologies, satisfiability, and logical consequence; semantic consequences; models; logical equivalence; applications to analysing logical validity of arguments.

### §4. ANALYSIS AND MANIPULATION OF PROPOSITIONAL LANGUAGE:

Truth functions; disjunctive and conjunctive normal forms; adequate sets of connectives; Cook's Theorem.

### §5. BOOLEAN ALGEBRAS:

Boolean algebras as algebras; Boolean algebras as orderings; natural examples of Boolean algebras; Boolean algebras in logic; the structure of Boolean algebras; isomorphism and set Boolean algebras.

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### §6. FORMAL PROOFS:

Computability, and formal proofs in propositional logic; natural deduction — the consequence relation, notation and rules; how to find formal proofs; soundness of natural deduction rules.

### §7. COMPLETENESS AND COMPACTNESS FOR PROPOSITIONAL LOGIC:

What we mean by completeness, consistency; Lindenbaum's lemma; a completeness proof for propositional logic; the compactness of propositional logic, and applications.

### §8. THE BEGINNINGS OF PREDICATE LOGIC:

Quantifiiers and first order logic; describing mathematical structures, and expressing everyday sentences in formal language; the difference between first and second order formulas; the symbols, terms and formulae of predicate logic.

### §9. INTERPRETATIONS:

Free and bound variables; sentences; interpreting predicate formulas; models, logical consequences; universally valid formulae; Prenex normal forms; some natural examples of first order structures. models.

### §10. FORMAL PROOFS, COMPLETENESS AND COMPACTNESS:

Rules for quantifiers, and formal proofs; outline of completeness and compactness for first order logic.

**NOTE: There will be FIVE sets of problems, and these will be an essential part of the course. The marks for these will contribute 15% to the final grade (3 marks for each set of problems).**

**In the final examination, NO CALCULATORS WILL BE ALLOWED.**