MATH 2040 MATHEMATICAL LOGIC I

Problems 2

SECTION A: Logical analysis of arguments

1. Comment on the validity of the following argument:
   If a graduate is well-qualified, he will get a job.
   The graduate is not well-qualified.
   Therefore he will not get a job.

2. Express each of the following arguments in propositional logic, and discuss their validity:
   (a) If the function \( f \) is not continuous, then the function \( g \) is differentiable. \( g \) is not differentiable. Therefore \( f \) is continuous.
   (b) If Smith has installed central heating, then either he has sold his car or he has borrowed money from the bank. Smith has not borrowed money from the bank. Therefore, if Smith has not sold his car, then he has not installed central heating.
   (c) If there is oil in Polygonia, then either the experts are right or the government is lying. There is no oil in Polygonia, or else the experts are wrong. Therefore the government is not lying.
   (d) If \( U \) is a subspace of \( V \), then \( U \) is a subset of \( V \), \( U \) contains the zero vector, and \( U \) is closed under addition. \( U \) is a subset of \( V \), and if \( U \) is closed under addition then \( U \) contains the zero vector. Therefore, if \( U \) is closed under addition, it is a subspace of \( V \).
   (e) Either a Republican or a Democrat is president. If a Democrat is president, then the Democrats control the Senate. Therefore the Democrats control the Senate.
   (f) If Mary lives in Leeds, then she lives in Yorkshire. If Mary lives in Leeds, then she lives in England. Mary lives in Yorkshire. Therefore she lives in England.

   continued overleaf →
SECTION B: Disjunctive normal form and adequate sets of connectives

1. Express the following formulae in disjunctive and conjunctive normal forms:
   (a) \((p \rightarrow q) \land (q \rightarrow p)\),
   (b) \((\neg(p \land q) \land \neg r) \rightarrow (\neg p \lor q)\).

2. Find propositional formulae (as simple as possible), in \(p\), \(q\), and \(r\), having the truth tables shown.

\[
\begin{array}{ccc|ccc}
 p & q & r & \text{(a)} & \text{(b)} & \text{(c)} \\
 T & T & T & T & F & T \\
 T & T & F & T & T & F \\
 T & F & T & T & F & T \\
 T & F & F & T & T & F \\
 F & T & T & T & T & T \\
 F & T & F & T & T & F \\
 F & F & T & T & F & T \\
 F & F & F & F & F & F \\
\end{array}
\]

3. Prove that the following sets of connectives are adequate:
   (a) \(\{\neg, \land\}\),
   (b) \(\{\text{nor}\}\).
   (Here \(\text{nor}\) is given by \(\varphi \text{ nor } \psi \equiv \neg(\varphi \lor \psi)\).)

4. Prove that the following sets of connectives are not adequate:
   (a) \(\{\land, \lor, \rightarrow\}\),
   (b) \(\{\neg, \leftrightarrow, \text{eor}\}\),
   (c) \(\{\text{cond}\}\).
   (Here \(\leftrightarrow\), \(\text{eor}\), \(\text{cond}\) are given by \(\varphi \leftrightarrow \psi \equiv (\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi)\), \(\varphi \text{ eor } \psi \equiv (\varphi \lor \psi) \land \neg(\varphi \land \psi)\) and \(\text{cond}(\varphi, \psi, \chi) \equiv (\varphi \rightarrow \psi) \land (\neg \varphi \rightarrow \chi)\): ‘if \(\varphi\) then \(\psi\) else \(\chi\)’.)

5. Find formulae \(\varphi\) and \(\psi\) logically equivalent to \(p \rightarrow q\) such that the only connective appearing in \(\varphi\) is \(\text{nand}\) and the only connective appearing in \(\psi\) is \(\text{nor}\). (Recall that \(\text{nand}\) is given by \(\varphi \text{ nand } \psi \equiv \neg(\varphi \land \psi)\).)

6. Prove that the only binary connectives \(\text{conn}\) such that \(\{\text{conn}\}\) is adequate are \(\text{nand}\) and \(\text{nor}\). (Hint; consider what the truth table of such \(\text{conn}\) would have to be.)

HAND IN SOLUTIONS TO **ALL** OF SECTION A, AND **THREE** QUESTIONS FROM SECTION B.