MATH-011101

This question paper consists of 3 printed pages, each of which is identified by the reference MATH-011101, with a 2-page Formula Sheet attached.

All calculators must carry an approval sticker issued by the School of Mathematics.

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Examination for the Module MATH–0111
(January 2014)

Elementary Differential Calculus

Time allowed: 2 hours

Attempt all questions in Section A and any three questions from Section B.

Each question in Section A carries 2 marks, each question in section B carries 20 marks.

You must show your working in your answers to all questions.

A formula sheet is supplied with this paper.

SECTION A

Attempt all the questions in Section A

A1. Expand \((3x - 2)(x - 5)\).

A2. Evaluate \(\left(\frac{1}{8}\right)^{2/3}\).

A3. Evaluate \(2a^3b^2a^{-1}b^{-1/2}\) when \(a = \frac{1}{2}\) and \(b = 4\).

A4. Find \(\log_8 16\).

A5. Factorise \(x^2 + 3x - 10\).

A6. Solve the equation \(x^2 + 6x - 7 = 0\).

A7. Find the equation of the straight line through the point \((-1, 1)\) which is perpendicular to the line \(2x - 3y + 1 = 0\).

A8. What is the distance between the points \((1, 2)\) and \((2, -2)\)?

A9. The angle \(\theta\) lies between 0 and \(\pi/2\) and \(\cos \theta = \frac{2}{5}\). Find \(\sin \theta\) and \(\tan \theta\) leaving your answers as exact expressions involving square roots.

A10. Find the equation of the circle with centre \((1, -1)\) and radius 3.
A11. Find $\frac{dy}{dx}$ when $y = x^{2/3}$.

A12. Find $\frac{dy}{dx}$ when $y = 2x^2 + x^3 + 8x$.

A13. Find $\frac{dy}{dx}$ when $y = \sqrt{2x + 3}$.

A14. Find $\frac{dy}{dx}$ when $y = \cos^3 x$.

A15. Find $\frac{dy}{dx}$ when $y = \frac{2x + 3}{x^3 + 2}$.

A16. Find $\frac{dy}{dx}$ when $y = e^{\pi x} \cos \sqrt{x}$.

A17. Find $\frac{dy}{dx}$ when $y = \ln(x^2 + \cos x)$.

A18. Find $\frac{d^2y}{dx^2}$ when $y = 2x^3 + 5x^2$.

A19. Find the tangent to the curve $y = 2x^2 - x + 3$ at the point $(1, 4)$.

A20. Without using a calculator, find an exact expression for $\cos(2\pi/3)$.

SECTION B

Attempt three questions in Section B

B1. (a) Sketch the graph of $y = \sin \theta$, for $\theta$ in the range $-2\pi \leq \theta \leq 2\pi$ labelling the values of $\theta$ where the graph crosses the horizontal axis and where $y$ has minimum and maximal values.

(b) Find all values of $\theta$ (in radians) between $-2\pi$ and $2\pi$, such that $\sin \theta = -\frac{1}{2}$.

(c) Using the formula for $\sin(A + B)$ from the formula sheet, show that $\sin(\theta + \frac{\pi}{2}) = \cos \theta$. Use the result from the previous part of the question to find all values of $\theta$ between $-\pi$ and $\pi$ such that $\cos \theta = -\frac{1}{2}$.

B2. (a) The points $A$, $B$ and $C$ have coordinates $(-1, 2)$, $(2, 3)$ and $(1, -2)$ respectively. Find:

(i) the equation of the line $AB$;
(ii) the equation of the line through $C$ perpendicular to $AB$;
(iii) the point where the above two lines meet;
(iv) the distance from $C$ to the line $AB$.
(b) A circle has centre at the point \(O = (2,1)\) and passes through the point \(P(-2, -3)\). Find:

(i) the radius of the circle;
(ii) the equation of the circle;
(iii) the gradient of the line \(OP\);
(iv) the equation of the tangent to the circle at \(P\).

B3. Differentiate each of the following functions with respect to \(x\).

(i) \(y = (x^4 - x)^2 + (x^4 - x)^{-1/2}\);
(ii) \(y = (2x^3 + \ln x) \cos(x - 1)\);
(iii) \(y = \frac{e^{2x} + 6}{x(x^2 + 4)}\);
(iv) \(y = \arccos(\ln x)\);
(v) \(y = \log_x \sqrt{x}\)

B4. (a) Find the stationary points of the function given by \(y = x^3 + x^2 - x + 1\) and determine whether they are (local) maximum or minimum points. Find the values of the function at these points.

(b) Find the maximum and minimum values of \(4 + 6x + 3x^2\) for \(x\) between \(-2\) and \(2\).

(c) If \(y\) is given as a function of \(x\) by \(2xy^2 + x^2y = \cos(x + 1)\), find \(\frac{dy}{dx}\) in terms of \(x\) and \(y\).