1. Start with the values \( \sin \frac{\pi}{6} = \frac{1}{2} \), \( \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \), and \( \sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \). Then

\[
\sin \frac{5\pi}{12} = \sin \frac{\pi}{6} \cos \frac{\pi}{4} + \cos \frac{\pi}{6} \sin \frac{\pi}{4} = \frac{1}{2} \cdot \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} = \frac{1 + \sqrt{3}}{2\sqrt{2}},
\]

and

\[
\cos \frac{5\pi}{12} = \cos \frac{\pi}{6} \cos \frac{\pi}{4} - \sin \frac{\pi}{6} \sin \frac{\pi}{4} = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}} = \frac{\sqrt{3} - 1}{2\sqrt{2}}.
\]

Hence

\[
\tan \frac{5\pi}{12} = \frac{\sin \frac{5\pi}{12}}{\cos \frac{5\pi}{12}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}.
\]

2. If we make the substitution \( \cos^2 \theta = 1 - \sin^2 \theta \), the equation \( 2 \cos^2 \theta = 2 - \sin \theta \) becomes \( 2 - 2 \sin^2 \theta = 2 - \sin \theta \), which simplifies to \( 2 \sin^2 \theta - \sin \theta = 0 \). This could be written \( 2s^2 - s = 0 \), where \( s = \sin \theta \), giving \( s(2s - 1) = 0 \). That is, \( \sin \theta = 0 \) or \( 1/2 \).

In the range specified, the solutions for \( \sin \theta = 0 \) are 0, \( \pi \) and \( 2\pi \), whereas for \( \sin \theta = 1/2 \) we have \( \pi/6 \) or \( 5\pi/6 \). These are the five solutions of the given equation.

3. Recall that \( \frac{1}{\cos^2 \theta} = \sec^2 \theta = 1 + \tan^2 \theta = 10 \), so \( \sec \theta = \sqrt{10} \) and \( \cos \theta = 1/\sqrt{10} \) (all the trigonometric functions are positive in this range). Thus \( \sin \theta = (\cos \theta)(\tan \theta) = 3/\sqrt{10} \).

Alternatively, draw a right-angled triangle ABC with a right angle at B and \( BC = 3 \), \( AB = 1 \). Then \( \tan A = 3 \), and the hypotenuse is \( AC = \sqrt{10} \), so \( \sin A = 3/\sqrt{10} \) and \( \cos A = 1/\sqrt{10} \).
4. and triangle for Q 3.

Table of values

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>$\pi/6$</th>
<th>$\pi/3$</th>
<th>$\pi/2$</th>
<th>$2\pi/3$</th>
<th>$5\pi/6$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cos 3x$</td>
<td>1</td>
<td>0</td>
<td>$-1$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

5. (a) Using the chain rule with $u = \cos x$ and $y = u^4$, we obtain $dy/dx = (4u^3)(-\sin x) = -4 \sin x \cos^3 x$.

(b) Using the chain rule with $u = 3x^2 + 4$ and $y = \sin u$, we obtain $dy/dx = (\cos u)(6x) = 6x \cos(3x^2 + 4)$.

(c) We have $y = (\sin x)^{-2}$, and the chain rule with $u = \sin x$ and $y = u^{-2}$ gives

$$
\frac{dy}{dx} = (-2u^{-3})(\cos x) = \frac{-2 \cos x}{\sin^3 x}.
$$

(d) By the quotient rule, $\frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2}$.

(e) Trick question: $y = 1$ (constant), so $dy/dx = 0$. Or you could differentiate each term using the chain rule and get $dy/dx = (2 \sin x)(\cos x) + (2 \cos x)(-\sin x) = 0$ again.

(f) The derivative of $\tan 2x$ is $2 \sec^2 2x$, and so by the product rule we get

$$
\frac{dy}{dx} = 2x \sec^2 2x + \tan 2x.
$$