School of Mathematics

MATH 0111 Elementary Differential Calculus

Solutions 7

1. For this function, \( \frac{dy}{dx} = 4x - 8 \), which is zero when \( x = 2 \). So \( y = 8 - 16 + 1 = -7 \).

Then \( \frac{d^2y}{dx^2} = 4 \), which is positive, and so we have a local minimum.

At \( x = 1 \), \( y = -5 \) and the gradient is \(-4\). So the tangent line is \( y + 5 = -4(x - 1) \), giving \( y = -4x - 1 \).

2. By the product rule (or you could multiply out first), \( \frac{dy}{dx} = 1 \cdot (7 - x) + (x + 3) \cdot (-1) = 4 - 2x \), which is zero when \( x = 2 \) and \( y = 5 \cdot 5 = 25 \). Then \( \frac{d^2y}{dx^2} = -2 \), which is negative, and so this is a local maximum.

At \( x = 1 \), \( y = 24 \) and the gradient is \( 2 \). So \( y - 24 = 2(x - 1) \), giving \( y = 2x + 22 \) as the tangent line.

3. Here, \( \frac{dy}{dx} = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x - 3)(x + 1) \), so the derivative vanishes at two points.

At \( x = 3 \), \( y = -26 \) and \( \frac{d^2y}{dx^2} = 6x - 6 = 12 \), which is positive. So this is a local minimum.

At \( x = -1 \), \( y = 6 \) and \( \frac{d^2y}{dx^2} = 6x - 6 = -12 \), which is negative. So this is a local maximum.

Finally, at \( x = 1 \), \( y = -10 \) and \( \frac{dy}{dx} = -12 \). So the tangent line is \( y + 10 = -12(x - 1) \), or \( y = -12x + 2 \).

4. Here, \( \frac{dy}{dx} = 7x^6 \) which vanishes only at \( x = 0 \), \( y = 0 \). The second derivative is \( 42x^5 \) which also vanishes, so we need to look more carefully. If \( x \) is small and positive then \( \frac{dy}{dx} > 0 \), and if \( x \) is small and negative then \( \frac{dy}{dx} > 0 \) again. This is therefore a point of inflexion.

At \( x = 1 \), \( y = 1 \) and \( \frac{dy}{dx} = 7 \). So the tangent is \( y - 1 = 7(x - 1) \), or \( y = 7x - 6 \).

5. Here, \( \frac{dy}{dx} = 4x^3 - 4 \), which vanishes only at \( x = 1 \), \( y = -3 \). The second derivative is \( 12x^2 \), which equals 12 and gives us a local minimum since it is positive.

The tangent has gradient 0 at \( x = 1 \), so its equation is \( y = -3 \) (a horizontal line).
Figure 1: Question 1

Figure 1b: Question 2

Figure 2: Question 3

Figure 2b: Question 4

Figure 3: Question 5