School of Mathematics

MATH 0111 Elementary Differential Calculus

Solutions 1

1. \((x - 4z)(2x - 3z^2) = (x - 4z)(2x) + (x - 4z)(-3z^2) = 2x^2 - 8xz - 3xz^2 + 12z^3\).

2. We have

\[(y - x)^3 = (y - x)(y - x)^2 = (y - x)(y^2 - 2xy + x^2) = (y - x)y^2 + (y - x)(-2xy) + (y - x)x^2 = y^3 - xy^2 - 2xy^2 + 2x^2y + x^2y - x^3 = y^3 - 3xy^2 + 3x^2y - x^3.\]

3. \((x^2 - 2y)(x^2 + 2y) = (x^2 - 2y)(x^2) + (x^2 - 2y)(2y) = x^4 - 2x^2y + 2x^2y - 4y^2 = x^4 - 4y^2\). You can see this more quickly as the “difference of two squares”.

4. \[\frac{2}{a - 1} + \frac{3}{a + 5} = \frac{2(a + 5) + 3(a - 1)}{(a - 1)(a + 5)} = \frac{5a + 7}{(a - 1)(a + 5)}.\]

5. \[\frac{y}{x + 2} + \frac{x}{y - 4} = \frac{y(y - 4) + x(x + 2)}{(x + 2)(y - 4)} = \frac{y^2 - 4y + x^2 + 2x}{(x + 2)(y - 4)}.\]

The technique in the next few questions is the same, so I’ll just do one in detail.

6. \(x^2 + 7x + 6\). The idea here is to find two numbers, say \(a\) and \(b\), such that \(a + b = 7\) and \(a \cdot b = 6\). Clearly we can take \(a = 6\), \(b = 1\), so \(x^2 + 7x + 6 = (x + 6)(x + 1)\). Check by multiplying out.

7. \(x^2 - 9x + 20 = (x - 5)(x - 4)\).

8. \(z^2 - 36 = (z - 6)(z + 6)\) (difference of two squares).

9. \(y^2 + y - 42 = (y + 7)(y - 6)\).

10. \(x^4 - 16x^2 = x^2(x^2 - 16) = x^2(x - 4)(x + 4)\). 

1
11. $15 + 8x + x^2 = (3 + x)(5 + x)$. If you are happier writing it as $x^2 + 8x + 15 = (x + 3)(x + 5)$, that is equally good.

12. $2x^3 - 4x^2 - 16x = 2x(x^2 - 2x - 8) = 2x(x - 4)(x + 2)$.

13. $5y^2 - 20 = 5(y^2 - 4) = 5(y - 2)(y + 2)$.

14. $x^2 - 10x + 21 = 0$ when $(x - 7)(x - 3) = 0$. So either $x - 7 = 0$ or $x - 3 = 0$. Thus $x = 7$ or $3$.

15. $x^2 - 14x + 49 = 0$ when $(x - 7)^2 = 0$. Thus $x = 7$ (a repeated root).

16. $a^2 + 4a = 5$ when $a^2 + 4a - 5 = 0$. So $(a + 5)(a - 1) = 0$. Thus $a = -5$ or $1$.

17. $z^2 + 10z + 16 = 0$ when $(z + 2)(z + 8) = 0$. Thus $z = -2$ or $-8$.

18. $x^3 - 2x^2 + x = 0$ when $x(x^2 - 2x + 1) = 0$. So $x(x - 1)^2 = 0$. Thus $x = 0$ or $1$.

19. $2z^2 - 12z + 18 = 0$ when $2(z^2 - 6z + 9) = 0$. So $2(z - 3)^2 = 0$. Thus $z = 3$ (only).